Past, Present, and Future: Developmental Mathematics

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Connecting past ←→ future

- "Memories are the key not to the past, but to the future." (Corrie Ten Boom)
- "We are made wise not by the recollection of our past, but by the responsibility for our future." (George Bernard Shaw)
- "People don't realize that the future is just now, but later." (Russell Brand)

It’s the Mathematics, silly!

Start with “Now” … 2018 !

- Minimization: smaller footprint for developmental mathematics
- Trend A: Co-requisite remediation (footprint size=0)
- Trend B: Pathways (smaller footprint for sub-populations)
- Trend C: Replace traditional dev math with modern courses (smaller footprint for all)
- Everybody is an expert (even college presidents and system chancellors)

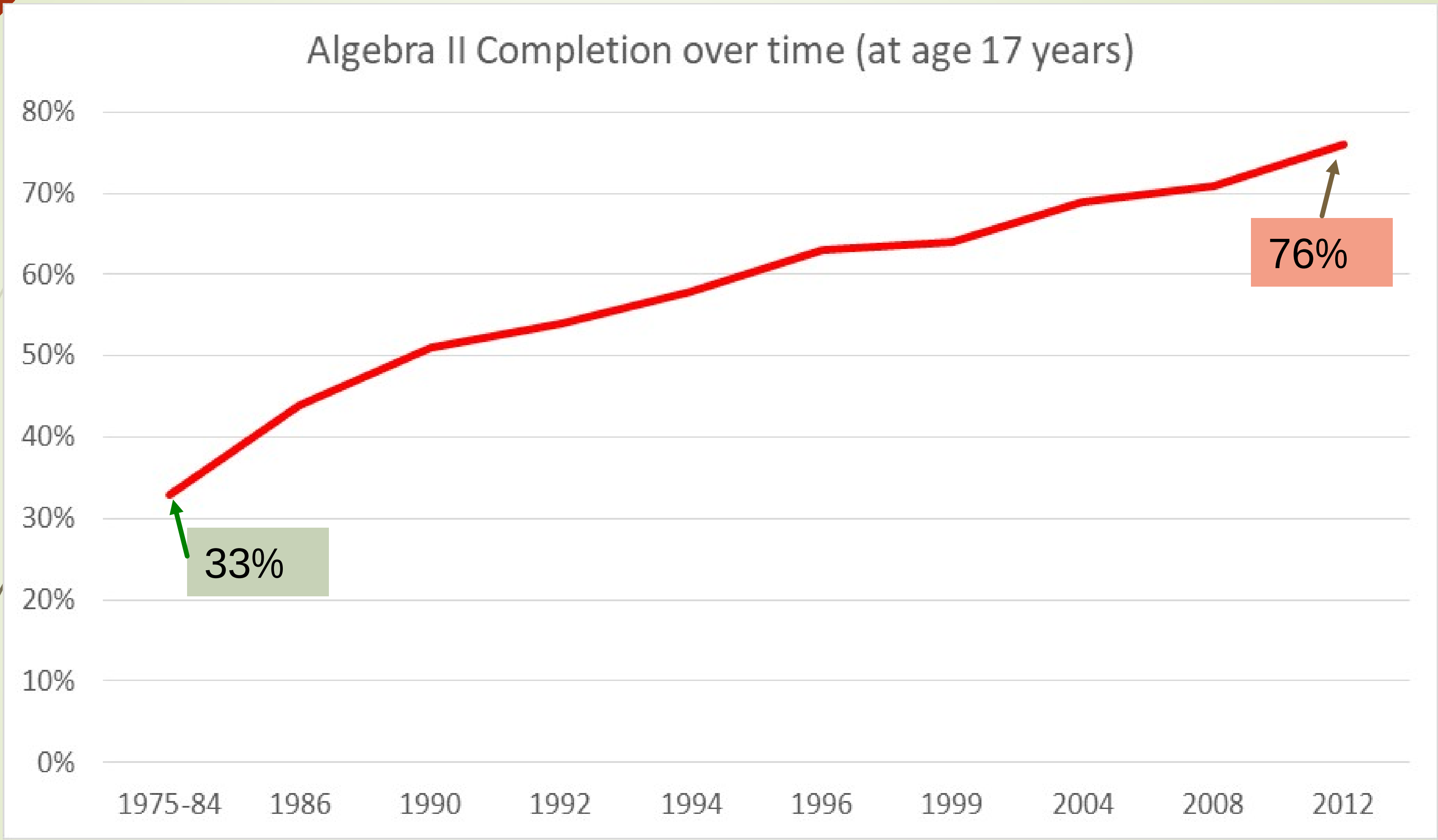
Poll: Which option do you prefer?

- A: Corequisite remediation
- B: Pathways
- C: Replace traditional courses
- D: None of these

In the beginning …

- Developmental mathematics … kinder, gentler remedial mathematics
- Complete the college-prep mathematics from high school, for those who did not
- “High school” mathematics cloned
- Was anti “New Math” (in general)
- Rationale: Get students ready for College Algebra or equivalent

Made some sense then (1975) ...



Bureau of Labor Statistics, "High school math courses and college attendance in two generations" http://www.bls.gov/opub/ted/2012/ted_20121016.htm
NCES Fast Facts: Advanced Mathematics & Science https://nces.ed.gov/fastfacts/display.asp?id=97
Brookings.edu: https://www.brookings.edu/research/algebra-ii-and-the-declining-significance-of-coursetaking/

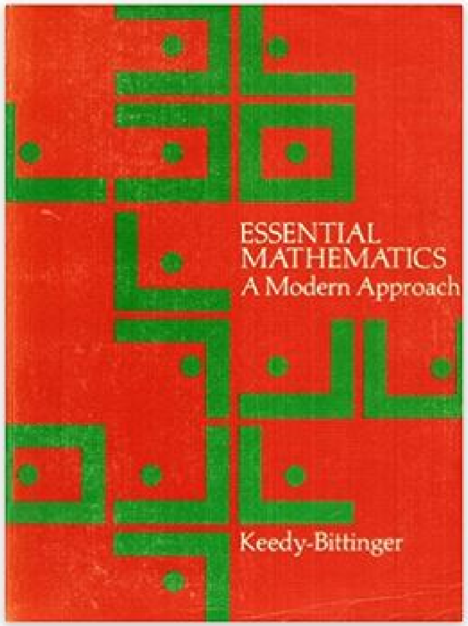
The Dev Math Curriculum … 1975

- Basic Math (<8th grade)
- Pre-Algebra (8th grade)
- Beginning Algebra (9th grade)
- Intermediate Algebra (11th or 10th grade)
- Some had Geometry (10th or 11th grade)

What we tried then (1975)

- Workbooks
- Programmed instruction books and specialized learning machines
- Audio tapes
- Books in 3 colors
- It was all about the **materials**

A sample ...



Essential Mathematics: Modern Approach Paperback – Import, December, 1972

by Mervin L. Keedy (Author), Marvin L. Bittinger (Author)

Be the first to review this item

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Hardcover	Paperback
from $50.00	from $49.99
2 Used from $50.00	1 Used from $49.99

This book contains a treatment of arithmetic, algebra, and trigonometry, and an appendix on the slide rule. The text adapts to either review or first time coverage and is designed mainly for a junior college audience with no prerequisites necessary. Practical application is emphasized throughout, with a minimum of words used to explain concepts; this stress on demonstration over description is especially helpful for remedial students. Of particular interest is the textbook page with wide margins intended for exercises a feature that creates an opportunity for student reinforcement and eliminates the cost of an extra workbook.

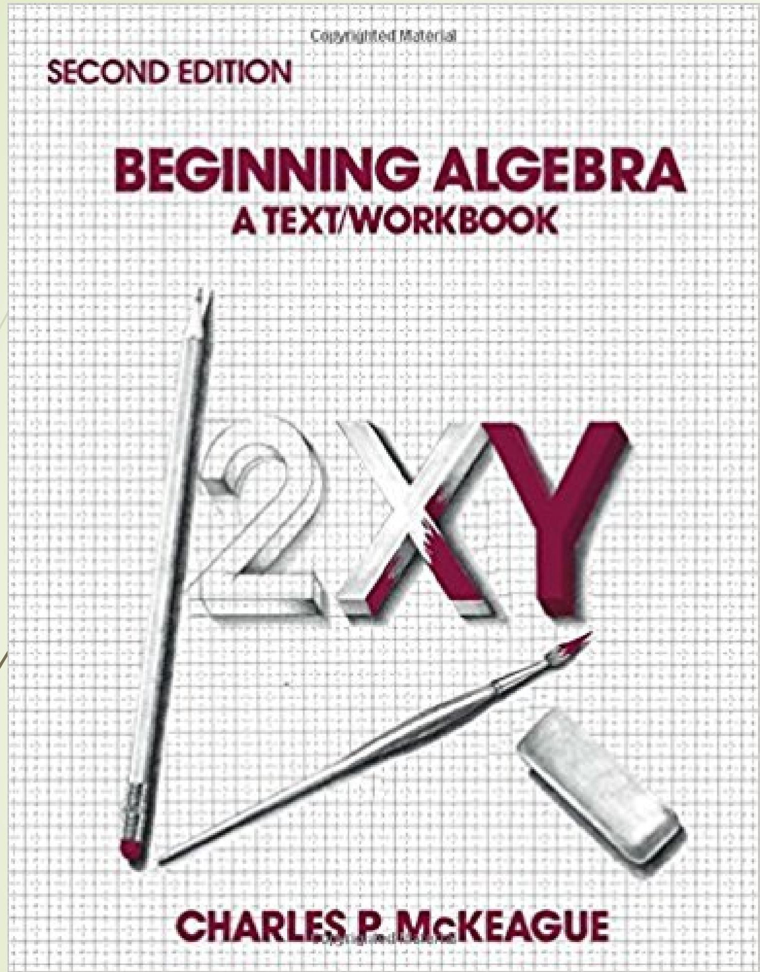
"Slide Rule" was the computing device of the era.

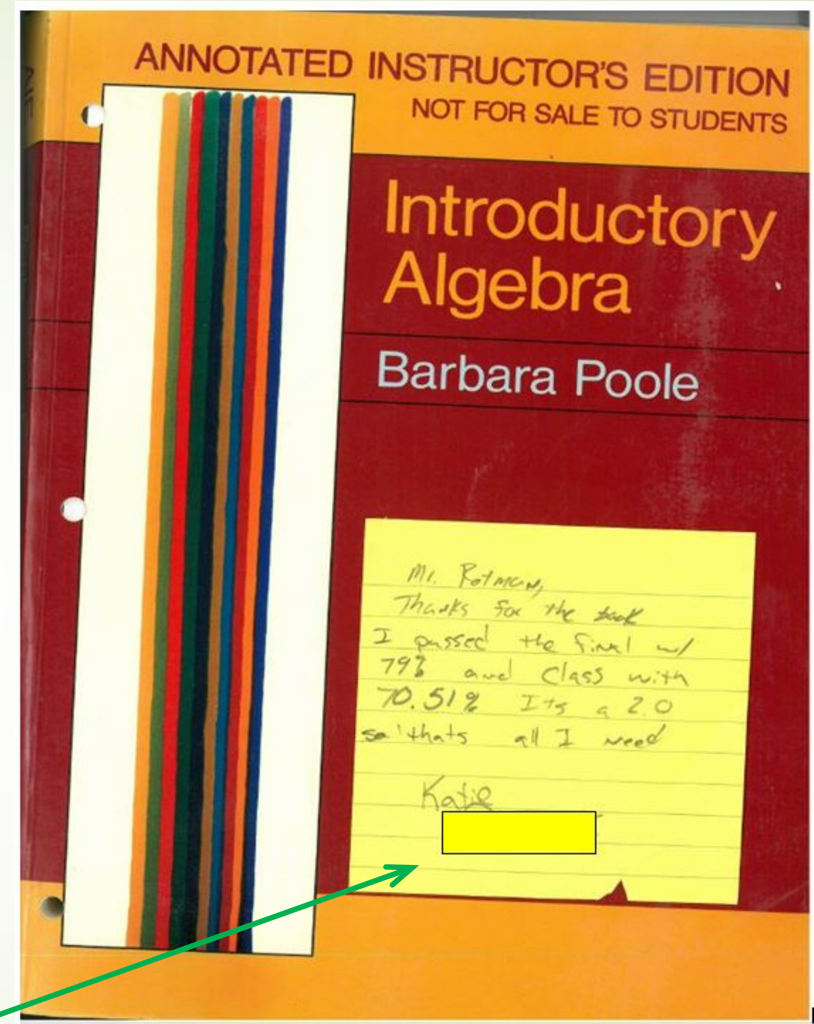
"Minimum of words" was a goal in many textbooks of the day.

The 1980's

- Arithmetic skills obsession (reaction to 'handheld calculators')
- Low pass rates meant "let's add another course!"
- Student Learning Problems (aka "blame the student")
- Back to Basics (skills, procedures)

Samples from the 1980's:





"Mr. Rotman, Thanks for the book. I passed the final w/79% and class with 70.51%. It's a 2.0 so that's all I need."

A Typical Section …

5.3 MULTIPLICATION OF POLYNOMIALS

1 Multiplying Monomials

Recall from Section 2.7 that when we multiply two exponential numbers with the same base, we keep the base and add the exponents.

$$a^m \cdot b^m = a^{m+n}$$

> To multiply two monomials multiply the coefficients and add the exponents on the identical variables.

EXAMPLE 1 Multiply.

a. $5(4x^2) = 5(4)(x^2) = 20x^2$

b. $3x(5x) = (3)(5)(x)(x) = 15x^2$

c. $(-6x^2y)(2x^3z^3) = (-6)(2)(x^2y)(x^3z^3) = -12x^5yz^3$

d. $-6ab(4a^2b^3) = (-6)(4)(ab)(a^2b^3) = -24a^3b^4$ Recall $ab = a^1b^1$ ■

□ DO EXERCISE 1.

2 Multiplying a Monomial and Any Polynomial

To multiply a monomial times any polynomial, multiply each term of the polynomial by the monomial. This is using a distributive law. Sometimes we apply it to more than two terms.

EXAMPLE 2 Multiply.

a. $4x(x^2 + 3) = 4x(x^2) + 4x(3)$ Using a distributive law

$= 4x^3 + 12x$

b. $-2y(y^2 + 4y - 7)$

$= -2y(y^2) + (-2y)(4y) - (-2y)(7)$ Distributive law for three terms

$= -2y^3 - 8y^2 + 14y$

Recall that a distributive law may be stated as $(b + c)a = ba + ca$.

c. $(x^2 - 2x + 3)3x = x^2(3x) - 2x(3x) + 3(3x) = 3x^3 - 6x^2 + 9x$ ■

□ DO EXERCISE 2.

OBJECTIVES

1. *Multiply monomials*
2. *Multiply a monomial and any polynomial*
3. *Multiply two polynomials*
4. *Multiply a binomial by a binomial using FOIL*

□ **Exercise 1** Multiply.

a. $-7(3y)$ — $-21y$

b. $-x(4x^2)$ — $-4x^3$

c. $8xy^2(-2x^3y^5)$ — $-16x^4y^7$

d. $-x(-8x^6y^4)$ — $8x^7y^4$

□ **Exercise 2** Multiply.

a. $-2x^2(x + 4)$ — $-2x^3 - 8x^2$

b. $3y(2y^2 - 4y + 8)$ — $6y^3 - 12y^2 + 24y$

c. $5y(y^3 - 7)$ — $5y^4 - 35y$

d. $-x(-x^2 + 3x - 1)$

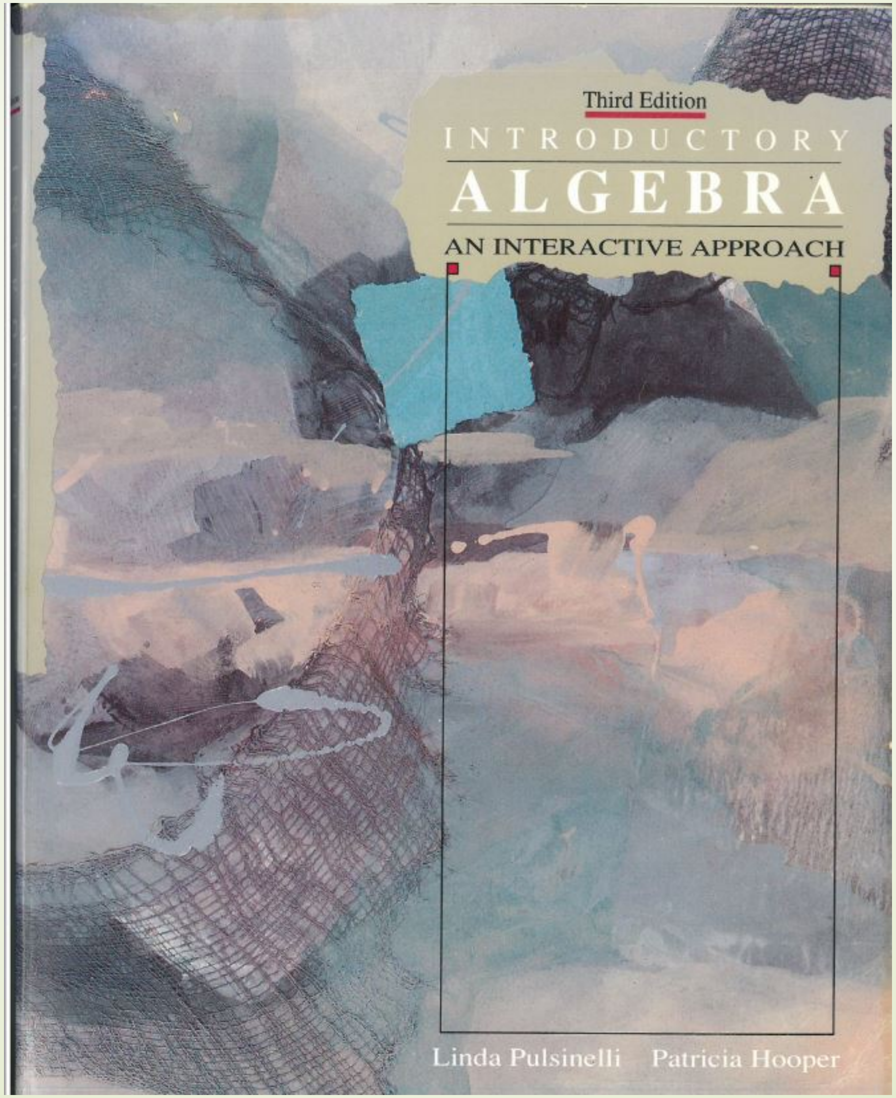
Text = Declarative Statement + Examples + Practice

De-emphasis of connections in content

The Early 1990s

- NCTM Standards … small changes for us
- Graphing calculators … all or nothing [Most of us did 'nothing']
- "Time for a change" (Ed Laughbaum)
- Many of the same messages then … as in "Common Vision" & "Math Sciences 2025"
- We still focused on: old curriculum, getting students ready for College Algebra

Sample from early 1990s



A Typical Section …

6.2 Multiplying with Monomials

Multiplying Monomials

In Chapter 5 we learned to use the First Law of Exponents to multiply powers of the same base. Recall that $a^m \cdot a^n = a^{m+n}$. We use the commutative and associative properties of multiplication when multiplying monomials containing constants *and* variables.

Example 1. Find $2x^3(4x^2)$.

Solution

$$\begin{aligned} 2x^3(4x^2) &= 2 \cdot x^3 \cdot 4 \cdot x^2 \\ &= 2 \cdot 4 \cdot x^3 \cdot x^2 \\ &= 8 \cdot x^{3+2} \\ &= 8x^5 \end{aligned}$$

You try to complete Example 2.

Example 2. Find $-5y^8(6y^7)$.

Solution

$$\begin{aligned} -5y^8(6y^7) &= -5 \cdot y^8 \cdot 6 \cdot y^7 \\ &= -5 \cdot 6 \cdot y^8 \cdot y^7 \end{aligned}$$

Check your work on page 281. ▶

To multiply monomials, first multiply the numerical coefficients, then multiply the variables using the First Law of Exponents.

Example 3. Find $4a^5\left(\frac{1}{3}a^7\right)$.

Solution

$$\begin{aligned} 4a^5\left(\frac{1}{3}a^7\right) &= 4 \cdot \frac{1}{3} \cdot a^5 \cdot a^7 \\ &= \frac{4}{3} \cdot a^{5+7} \\ &= \frac{4}{3}a^{12} \end{aligned}$$

You complete Example 4.

Example 4. Find $3x^2(-2x^4)(-7x)$.

Solution

$$\begin{aligned} &3x^2(-2x^4)(-7x) \\ &= 3(-2)(-7) \cdot x^2 \cdot x^4 \cdot x \end{aligned}$$

Check your work on page 281. ▶

Example 5. Find $0.8x(3x^2y)$.

Solution

$$\begin{aligned} &0.8x(3x^2y) \\ &= 0.8 \cdot 3 \cdot x \cdot x^2 \cdot y \\ &= 2.4x^3y \end{aligned}$$

Try to complete Example 6.

Example 6. Find $(-2xy^2z)(-9x^2y^5z^7)$.

Solution

$$\begin{aligned} &(-2xy^2z)(-9x^2y^5z^7) \\ &= (-2)(-9) \cdot x \cdot x^2 \cdot y^2 \cdot y^5 \cdot z \cdot z^7 \\ &= \underline{\qquad\qquad} \end{aligned}$$

Check your work on page 282. ▶

STILL:
Text = Declarative Statement + Examples + Practice

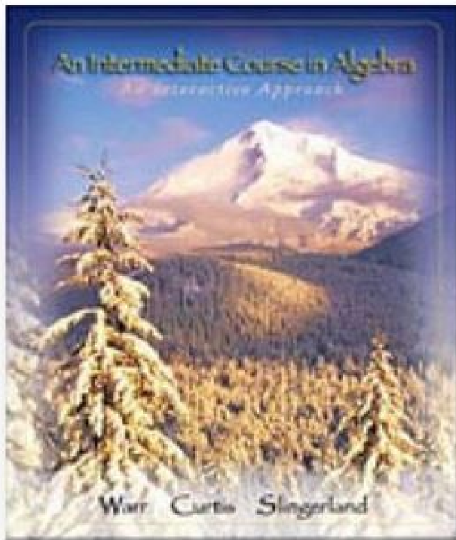
The Late 1990s

- Pockets of reform and revolution:
  Focus on writing textbook(s); some grant based
- Supported by AMATYC Standards (1995) and NCTM standards (though not by 'us')
- Presentations at AMATYC and affiliates
- Some efforts were similar to current "Option C": Replace traditional dev math with modern courses

One of the 1990s Reform Books

An Intermediate Course in Algebra: An Interactive Approach (with InfoTrac) / Edition 1

by Alison Warr, Cathy Curtis, Penny Slingerland



ISBN-10: 0534436730

Hardcover

$12.64

ADD TO CART

Condition:Good

Sold by SellBackYourBook

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Seller Rating ★★★★½ (2419)

Seller Comments:

0534436730 Item in good condition. Textbooks may not include supplemental items i.e. CDs, access codes etc... All day low prices, buy fro[m] us sell to us we do it all!!

"This book was written to address the challenge of the NCTM and AMATYC Standards and technology integration in the classroom. The authors address the standards using a variety of methods, including Numerical, Graphical, and Algebraic Models; Guided Discovery Activities; Problem Solving; Technology; Collaborative Learning."

Another sample (1990s reform)

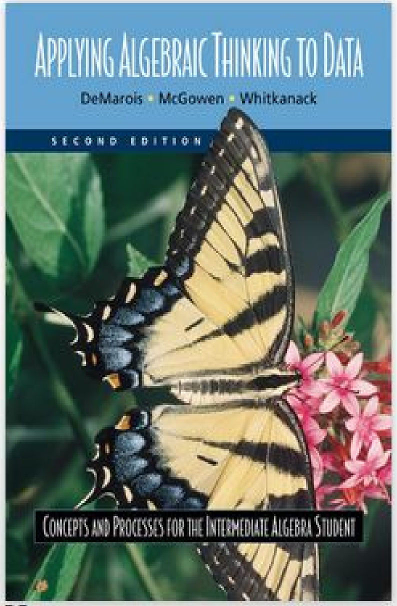
Applying Algebraic Thinking to Data: Concepts and Processes for the Intermediate Algebra Student, 2nd Edition

Phil DeMarois, Mt. Hood Community College

Mercedes McGowen, William Rainey Harper College

Darlene Whitkanack, University of Illinois, Chicago

©2001 | Pearson | Out of print

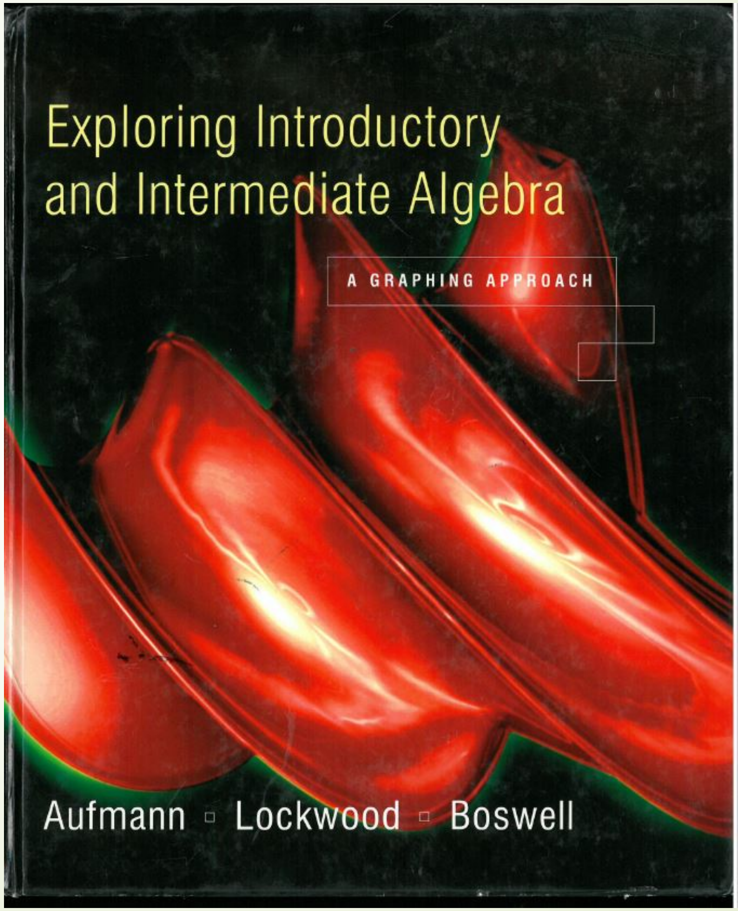


View Larger

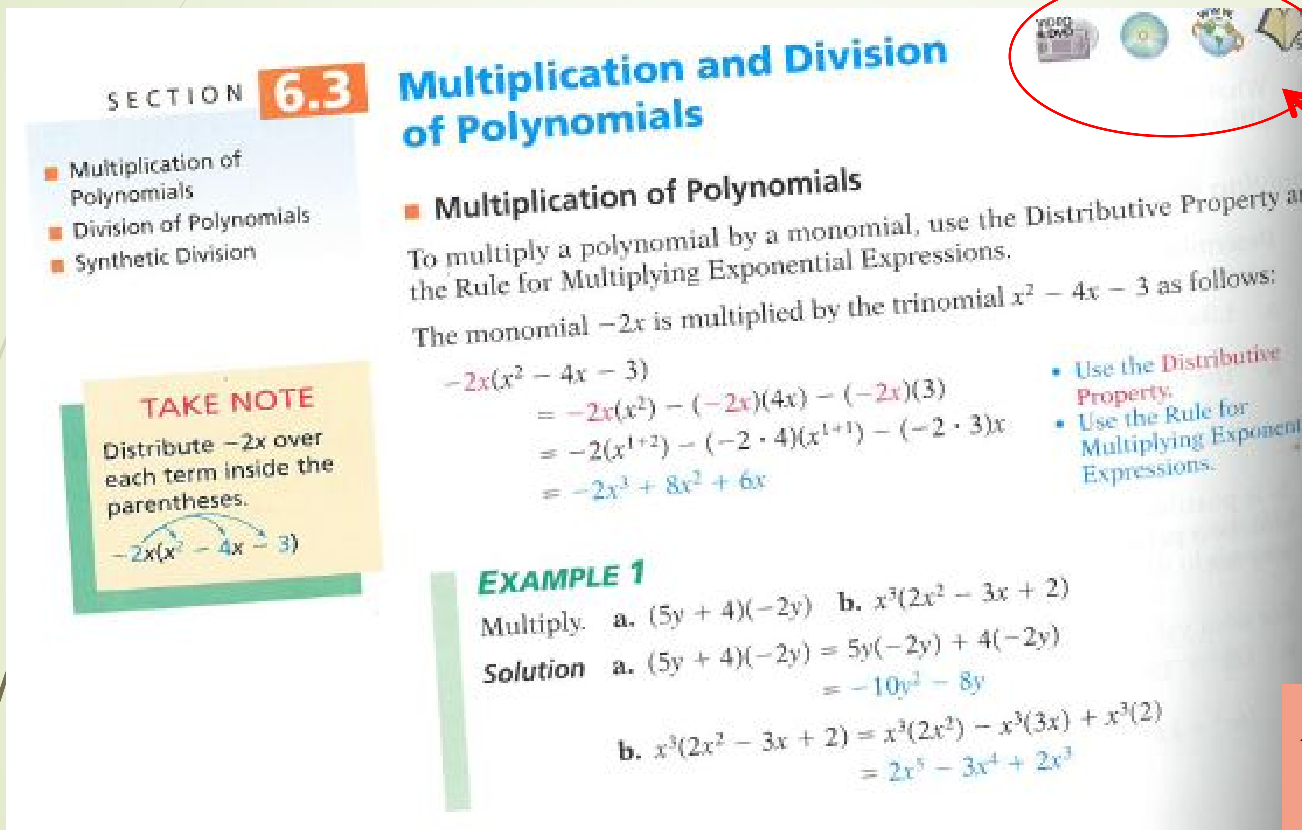
2000 to 2009

- Publisher's Golden Age: lots happening
- Digital as supplement
- Focus on commonly used content
- Reduction in reform books, and growth of combined algebra texts
- Separate and unequal: graphing calculator within some textbooks; most avoid GC
- Few of us thought of anything besides College Algebra

Text sample … 2004



A Typical Section



SECTION 6.3

Multiplication and Division of Polynomials

- Multiplication of Polynomials
- Division of Polynomials
- Synthetic Division

Multiplication of Polynomials

To multiply a polynomial by a monomial, use the Distributive Property an[d] the Rule for Multiplying Exponential Expressions.

The monomial $-2x$ is multiplied by the trinomial $x^2 - 4x - 3$ as follows:

$$\begin{aligned} &-2x(x^2 - 4x - 3) \\ &= -2x(x^2) - (-2x)(4x) - (-2x)(3) \\ &= -2(x^{1+2}) - (-2 \cdot 4)(x^{1+1}) - (-2 \cdot 3)x \\ &= -2x^3 + 8x^2 + 6x \end{aligned}$$

- Use the Distributive Property.
- Use the Rule for Multiplying Exponenti[al] Expressions.

TAKE NOTE

Distribute $-2x$ over each term inside the parentheses.

$-2x(x^2 - 4x - 3)$

EXAMPLE 1

Multiply. a. $(5y + 4)(-2y)$ b. $x^3(2x^2 - 3x + 2)$

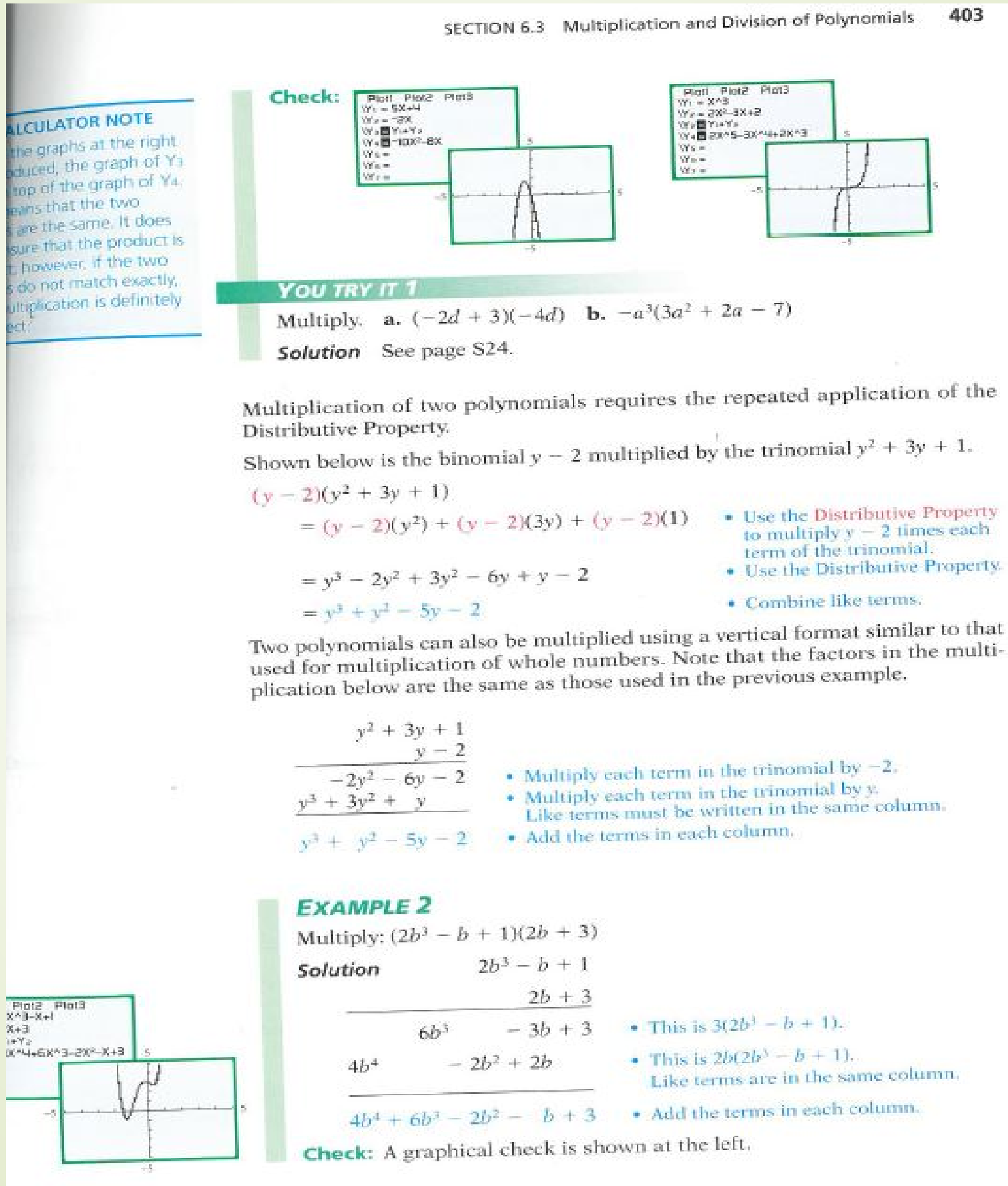
Solution a. $(5y + 4)(-2y) = 5y(-2y) + 4(-2y)$
$= -10y^2 - 8y$

b. $x^3(2x^2 - 3x + 2) = x^3(2x^2) - x^3(3x) + x^3(2)$
$= 2x^5 - 3x^4 + 2x^3$

Digital as supplement

And still:
Text =Declarative Statement + Examples + Practice

A Typical Section



ALCULATOR NOTE
the graphs at the right
oduced, the graph of Y_3
top of the graph of Y_4,
eans that the two
s are the same. It does
sure that the product is
t; however, if the two
s do not match exactly,
ultiplication is definitely
ect.

Check:

YOU TRY IT 1

Multiply. **a.** $(-2d + 3)(-4d)$ **b.** $-a^3(3a^2 + 2a - 7)$

Solution See page S24.

Multiplication of two polynomials requires the repeated application of the Distributive Property.

Shown below is the binomial $y - 2$ multiplied by the trinomial $y^2 + 3y + 1$.

$(y - 2)(y^2 + 3y + 1)$

$= (y - 2)(y^2) + (y - 2)(3y) + (y - 2)(1)$ • Use the Distributive Property to multiply $y - 2$ times each term of the trinomial.

$= y^3 - 2y^2 + 3y^2 - 6y + y - 2$ • Use the Distributive Property.

$= y^3 + y^2 - 5y - 2$ • Combine like terms.

Two polynomials can also be multiplied using a vertical format similar to that used for multiplication of whole numbers. Note that the factors in the multiplication below are the same as those used in the previous example.

$$\begin{array}{r} y^2 + 3y + 1 \\ y - 2 \\ \hline -2y^2 - 6y - 2 \\ y^3 + 3y^2 + y \quad \\ \hline y^3 + y^2 - 5y - 2 \end{array}$$

• Multiply each term in the trinomial by −2.
• Multiply each term in the trinomial by y. Like terms must be written in the same column.
• Add the terms in each column.

EXAMPLE 2

Multiply: $(2b^3 - b + 1)(2b + 3)$

Solution

$$\begin{array}{r} 2b^3 - b + 1 \\ 2b + 3 \\ \hline 6b^3 - 3b + 3 \\ 4b^4 - 2b^2 + 2b \quad \\ \hline 4b^4 + 6b^3 - 2b^2 - b + 3 \end{array}$$

• This is $3(2b^3 - b + 1)$.
• This is $2b(2b^3 - b + 1)$. Like terms are in the same column.
• Add the terms in each column.

Check: A graphical check is shown at the left.

Graphing calculator: Numeric methods to check symbolic procedures

Next:
AMATYC Standards, Act 2

- Beyond Crossroads (2006)
- Process as a Focus ("Improvement Cycle")
- Curriculum addressed more in 1995 document
- Implicit acceptance of status quo (the out-of-date remediation structure)
- Policy influencers … began to be interested in developmental mathematics

2000-2009: NCAT

- The National Center for Academic Transformation
- Course Redesign as the all-purpose solution:
  Emporium; Modules
- Skills … old content
- Efficiency
- Generality: Isolated from the work of the profession

It’s still the Mathematics!

The Role of 2010

- Carnegie Foundation: Quantway™ and Statway™
- Dana Center: Foundations of Mathematical Reasoning
- AMATYC New Life: Mathematical Literacy; Algebraic Literacy (the forgotten sibling)
- The "joyful conspiracy" (Uri Treisman)
- We began thinking about other college math courses (besides 'college algebra')

No Longer Hidden

- Prior to 2010, dev math operated under the radar
- Until … Policy influencers painted a dismal picture of our work
- Policy influencers sought to disrupt the continuity in the profession
- Specific solutions “sold” to college and system leaders (presidents, provosts)
- Focus on non- (or anti-) College Algebra

Minimization Option A: Footprint=0

- Co-requisite remediation as the all-purpose solution
- Focus on Statistics & Liberal Arts Math (or QR)
- “The data is in … co-requisite remediation works”
- “We can’t a group of students for which it does not work.” If it sounds too good to be true … is it?
- **College algebra de-valued; get done with math!**

Minimization Option B: Some Gain

- Pathways: Math Literacy replaces 1 or 2 algebra courses for SOME students
- Students needing statistics or quantitative reasoning (aka “non-STEM”)
  arithmetic courses often still required;
- “STEM” students generally see the same old curriculum **(obsolete stuff)**
- ***Algebra Avoidance as institutional policy***

Minimization Option C: Replacement

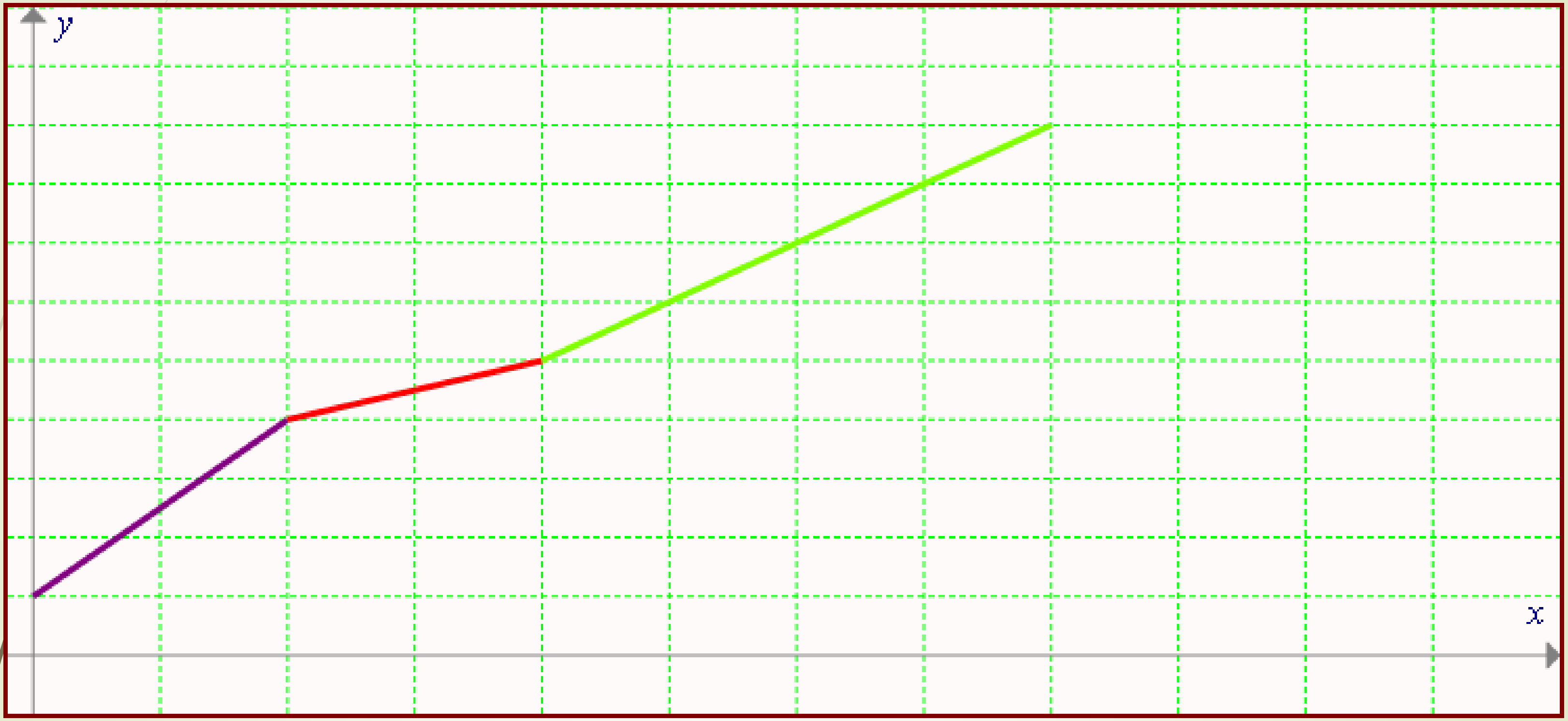
- **Mathematical needs: “same” for all students at the Math Literacy level**
- Eliminate arithmetic (and pre-algebra)
- Intermediate Algebra is not appropriate today: Need “Algebraic Literacy”
- **Supports College Algebra as well as ‘other mathematics’** (stat, QR, etc)
- Supports upward mobility (mid- and high-skill technical programs)

The College Mathematics Curriculum

- Minimization also applies to college level math courses
- Obsolete content: will become modern, efficient
- Continuity is critical … our values, our dreams for ‘better’
- **“Replacement” (option C) is our first step towards improving ALL of our courses**

MAA CUPM 2015, “The Calculus Sequence”
https://www.maa.org/sites/default/files/CalculusCASGReportFinal.pdf
National Academy Press, “The Mathematical Sciences in 2025”
https://www.nap.edu/catalog/15269/the-mathematical-sciences-in-2025
MAA, “Common Vision for Undergraduate Mathematical Sciences Programs in 2025”
https://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf

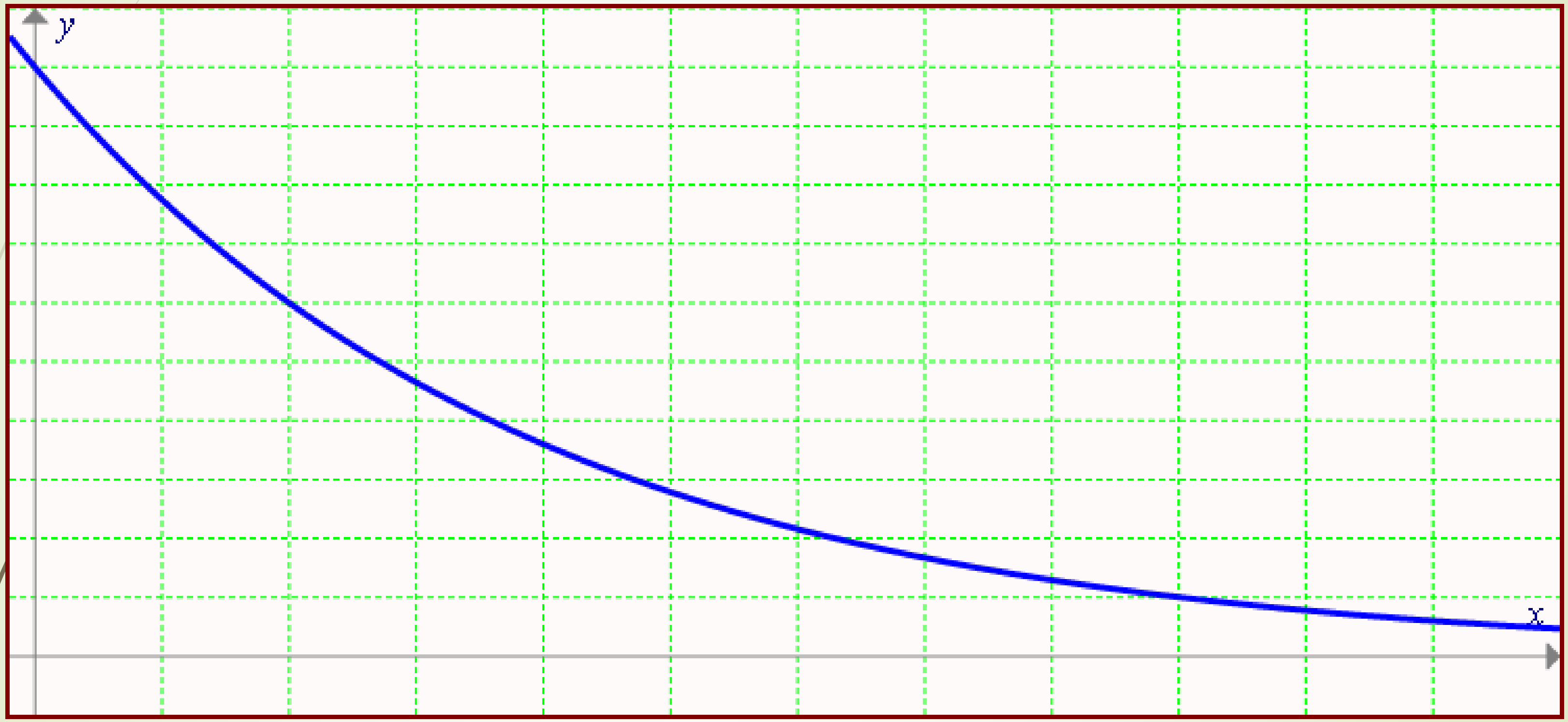
What WE see



“More developmental courses leads to more students being ‘ready’!”

What THEY see

"They" refers to policy influencers … such as Complete College America, Jobs for the Future, and others.

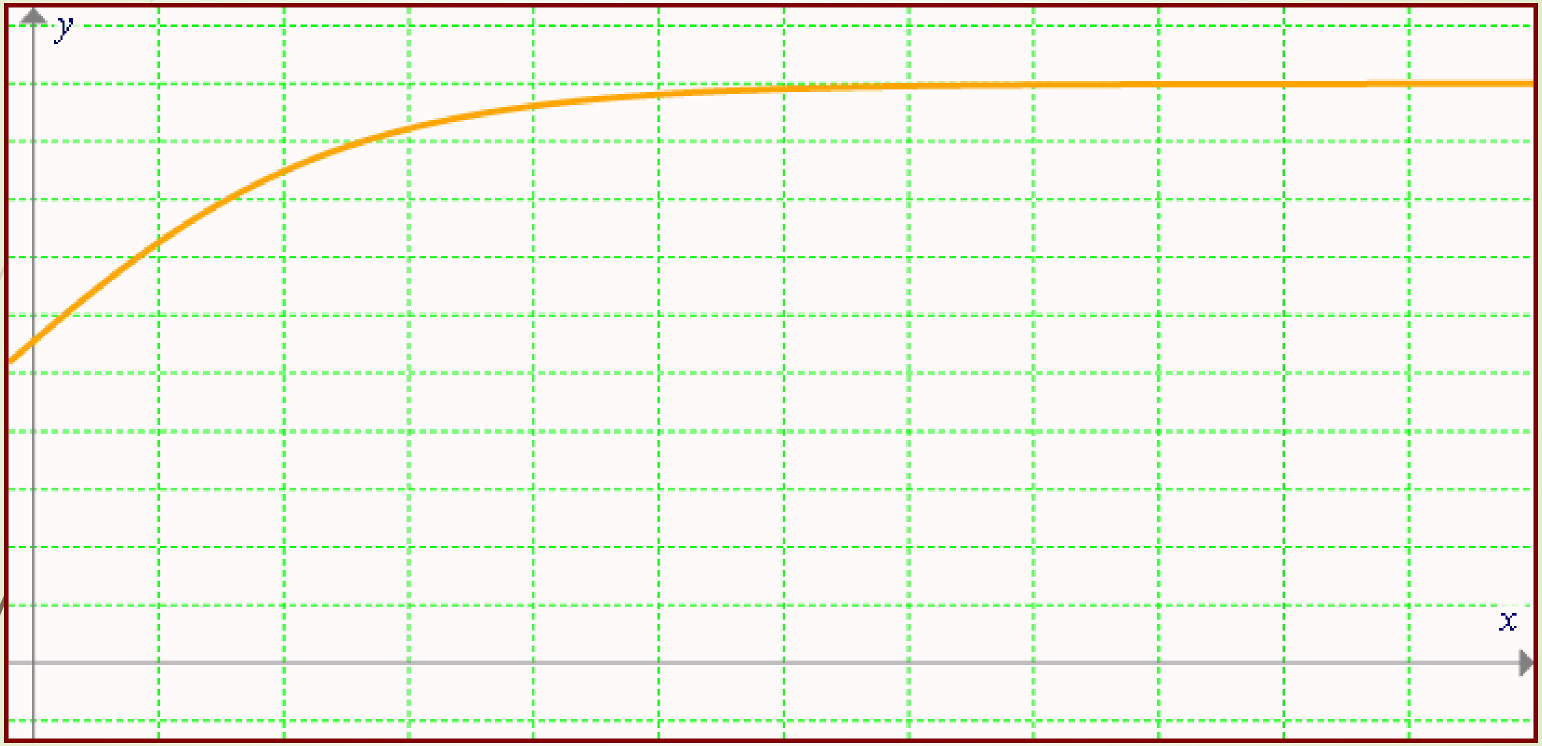


"More developmental courses means most students are blocked from completion!"

“Remediation” will not survive

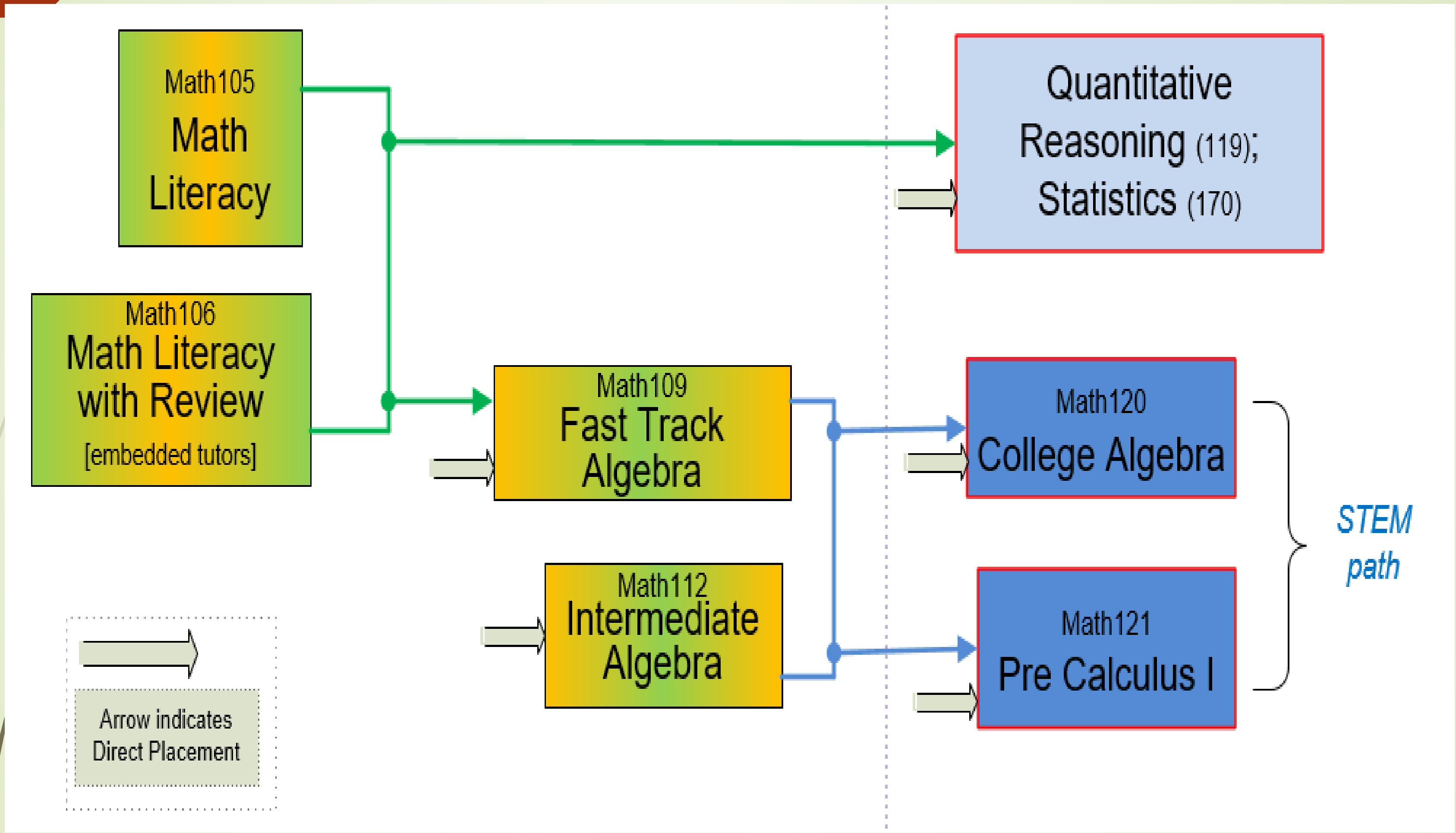
- Exponential decay is stronger: we can not WIN this argument
- Stop using the labels “remedial” and “developmental”
- Articulate a positive message about effective & modern preparation courses that we can show lead to success in ALL fields (not just non-STEM)
- Such as: **One (at most) pre-college prep course for 90% of students**

Our Future



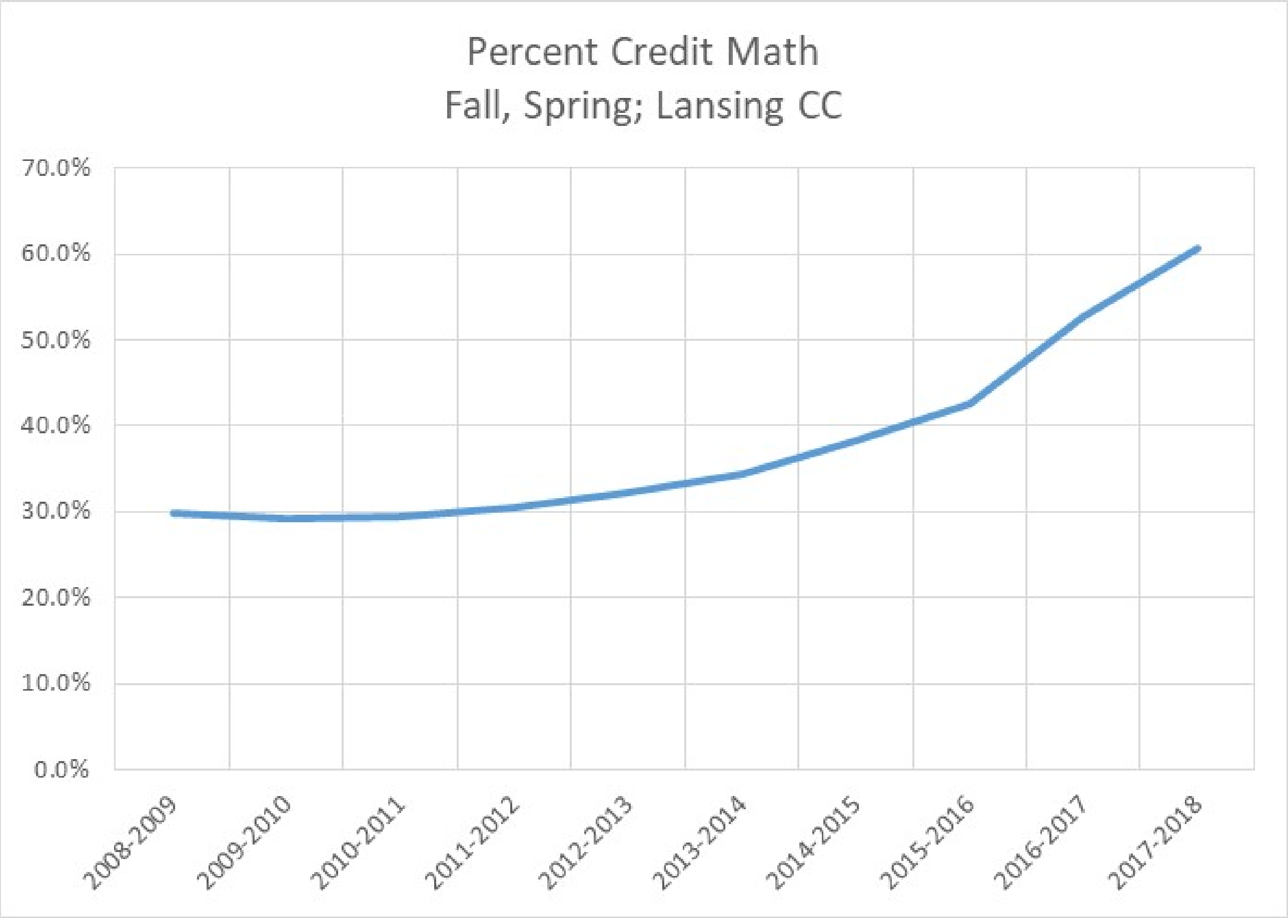
“One course gets 90% of students ready for success in college!”

What this looks like: Lansing CC



60% of current enrollment is in credit courses (up from 30%).

Credit course enrollment (math)



Curricular changes resulted in a doubling of this rate

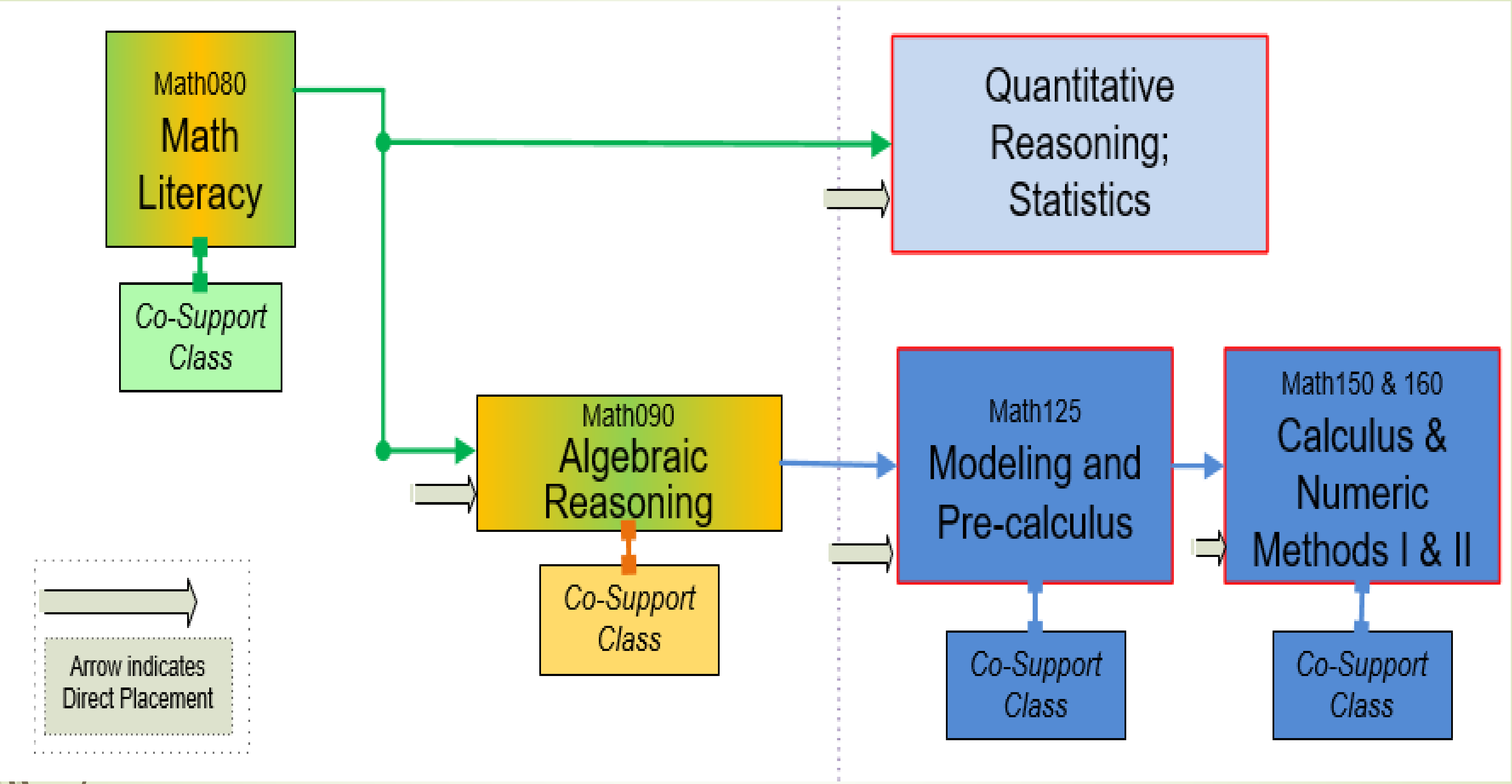
Poll:

Is this a reasonable goal …

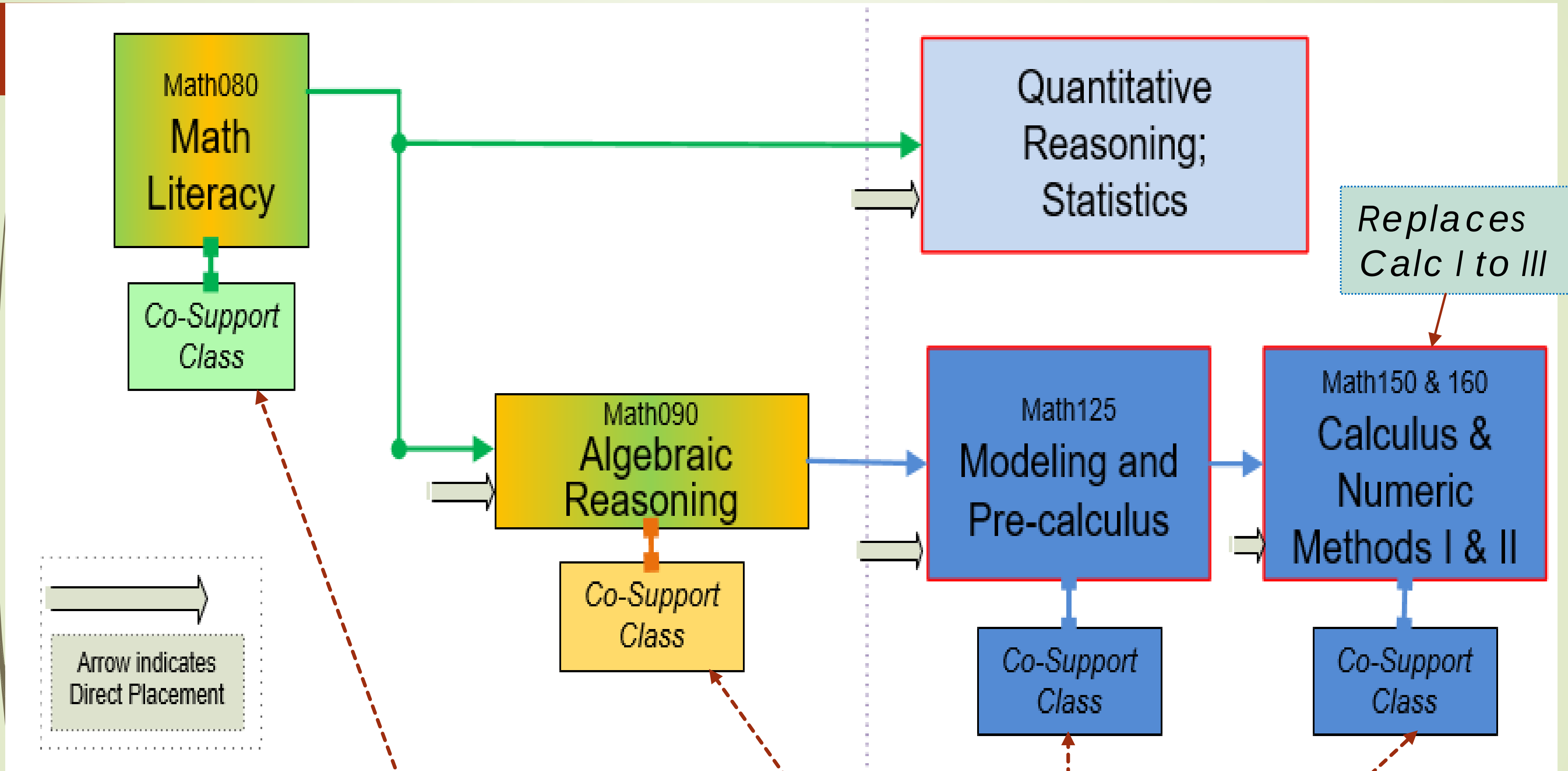
One course (at most) gets 90% of our students ready for College mathematics?

- Yes
- No

The Future Might Be … Generic



The Future Might Be …



Co-Support classes for select placement groups: lower 1/3 in Math Lit

For select groups: lower ¼ placement in this course, plus C & 2.5 grades in Prereq course

Where are we headed?

- All traditional developmental math courses will be gone within 5 years; several forces ensure that
- Survival of stand-alone “dev math” (prep) courses depends upon our professional work
- Co-requisite remediation will be an accepted solution. How will it serve our purposes?
- Intro college math courses (up to Calculus ***n***) are the next field of dreams: where do WE want to go?

- **It's still about the mathematics!!**

Closing

- Focus on what is important
- Progress is more important than change
- College-prep must reflect contemporary K-12 education (not the 1970s)
- Remember: external mandates will not last forever

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