CHAPTER 3 - A Faculty's Guide: Mathematics in the First Two Years

Faculty exert strong influences over student learning in mathematics and student success. In order to produce positive influences, faculty need to design a learning environment that fosters student engagement and empowers students to take ownership of their learning. Student success requires curriculum and courses that are designed to meet the needs of students, whether they are in STEM fields, or other fields, and that are aligned with the two-year college mission of career, responsible citizenry, and personal improvement. . Faculty also need to use student performance to assess effectiveness and make changes needed to assure that students are proficient, not only in performing routine exercises, but in meaningful analysis, modeling and problem solving.

Faculty are responsible for the critical components which build PROWESS within a modern college environment. Faculty have primary ownership of:

- Curriculum: Sets of courses designed to provide mathematics aligned with the best thinking about content
- Course design: Each course is developed to connect that mathematics to students and their learning, including delivery methods and learning technology
- Learning environment: Instruction and assessment practices intentionally developed to support all students
- Assessment and Maintenance: Improvement methodologies applied consistently to build long-term success

These responsibilities require that every faculty member develop both content expertise and knowledge of the cognitive sciences.

Faculty also work with others in these areas:

- Procedures that assist students in initial placement into a mathematic course
- Advising students as to which mathematics course is most appropriate
- Analyzing student, course, and program data

By “faculty” we mean all faculty engaged with teaching mathematics within the first two years of college, regardless of the specific role. The responsibilities are held both individually and collectively; this chapter will focus on individual faculty and some groups of faculty. The next chapter will address responsibilities of departments and institutions.

Faculty PROWESS begins with proficiency in curriculum and instruction within mathematics. Developing this proficiency is a long-term process based on engagement with professional organizations (including AMATYC, MAA, ASA, SIAM, and others) combined with local & regional resources for faculty development.

Standards in Curriculum

The AMATYC IMPACT standards are not focused on any particular curriculum or model. However, we do endorse some design principles to assist faculty with their responsibilities. The Common Vision document provides a framework for improving the curriculum in the first two years (pgs 12-13):

- ∑ Enhance students' perceptions of the beauty, vitality, and power of the mathematical sciences.
- ∑ Enhance students' understanding of mathematics as a creative endeavor.
- ∑ Increase students' quantitative and logical reasoning abilities needed for informed citizenship and for the workplace.
- ∑ Increase students' confidence and joy in doing mathematics and statistics
- ∑ Improve students' ability to communicate quantitative ideas orally and in writing (and since a precursor to communication is understanding, improve students' ability to interpret information, organize material, and reflect on results).
- ∑ Encourage students to continue taking courses in the mathematical sciences

Based on these goals, the AMATYC IMPACT standards provide the following general goals for our curriculum in the first two years of college mathematics:

- ➢ Faculty should have deep knowledge of recent professional work in curriculum relevant to their primary areas of responsibility (MAA CUPM … ASA GAISE … AMATYC IMPACT … and so on). This expertise is just as important as the initial credential required of faculty.
- ➢ Mathematics courses at all levels need to balance multiple approaches to reflect the status of the profession (such as symbolic and numeric methods). Computation & modeling skills using inductive reasoning should be developed concurrently with formal deductive reasoning.
- ➢ Since students will be expected to transfer what is learned in mathematics classrooms to a variety of contexts and situations, the curriculum needs to focus on reasoning at multiple levels within each mathematics course. In addition, students must be challenged to apply mathematics in a variety of problem situations. The phrase "quantitative literacy" is sometimes used to capture this goal.
- ➢ The curriculum sequence should be designed to support the needs of students in a wide variety of college programs. In some cases, this means 'pathways' choices should be included in the curriculum in which a sequence of courses is designed to serve only students in specific programs.

The chart below captures specific standards for parts of the curriculum.

Quantitative Literacy (All mathematics courses)	**Developmental Mathematics**	**General Education**
Determine appropriate quantitative literacy outcomes for each mathematics course and include these outcomes in course outlines Encourage students to represent and communicate mathematical ideas using a variety of modalities including numerical, graphical, symbolic, and verbal representations Promote mathematics across the curriculum by initiating collaborations with faculty with other disciplines, promoting the integration of quantitative literacy in all programs of the college, and promoting positive attitudes by all faculty and students towards mathematics Integrate technology into every mathematics course as both an aid in computation and as a modeling tool which encourages students to investigate patterns, formulate and test conjectures. Promote algebraic thinking through activities which emphasize the recognition of patterns, relations, and functions; reinforce the importance of the acquisition of skills in manipulating algebraic expressions Provide students with opportunities to apply a variety of problem-solving strategies to solve both routine and non-routine problems including substantive and contextualized situations Intentionally design instruction to help students build strong cognitive schema focusing on coherence, connections and understanding.	Design courses and classes to produce desired student outcomes in developmental mathematics within one year of college attendance, using accelerated formats for appropriate cohorts of students Make decisions about content (what to emphasize, de-emphasize, delete, or add) based on input from other professionals and organizations as well as in an effort to provide the most appropriate experience for all students Demonstrate and encourage multiple problem-solving strategies using appropriate tools from algebra, geometry, and statistics Integrate technology into developmental mathematics courses as a tool to investigate and promote understanding of mathematical concepts Use modern teaching methods including active learning and technology Ensure that understanding and reasoning are represented in the majority of course and program learning outcomes. Provide a variety of 'practice' to reinforce ideas, including blocked practice but emphasizing mixed practice for better learning	[Includes Quantitative Reasoning, Liberal Arts Mathematics, Statistics and other courses] Include writing, critical thinking, quantitative literacy, and logical reasoning needed for an informed citizenry Help students develop an understanding of the interconnectedness of mathematical ideas and how these ideas can be used to describe real-world phenomena Collaborate with faculty from other disciplines to determine learning outcomes for general education mathematics courses Use data with modeling or statistical tools to help students develop better understandings of economic, social or cultural issues

Technical & Occupational	Teacher Preparation	Mathematics Intensive
Establish partnerships with representatives from local businesses and industry to identify essential skills that students must bring to the workplace. Use their input to keep course content relevant Incorporate workplace skills in the learning outcomes of the curriculum (including 'soft skills') Encourage active student learning and the development of team-building skills with term projects, collaborative projects, portfolios, research, field investigations, or internships Use technology throughout curricula to discover properties, to develop concepts, to examine multiple perspectives, and to gain experience with the technology skills they will use for problem solving in the workplace Use data with modeling or statistical tools to help students develop better understandings of how mathematics is used a tool in other disciplines	Design standards-based courses that help students develop a deep understanding of the nature of mathematical concepts and mathematical thinking, encourage students to seek multiple ways to solve a problem, and make judicious use of technology, manipulatives, and other learning resources Use multiple assessment strategies to assess students Include supervised field experiences for future teachers in some of the mathematics courses Recruit students from underrepresented groups into the mathematics teaching profession. Use data with modeling or statistical tools to equip students with an understanding of how data and technology are used as tools in other mathematics courses and in other professions	Stress the pivotal roles of functions and modeling in solving real-world problems using both symbolic and numeric methods; favor the application of concepts over memorization Create courses and activities to promote student ability to use multiple approaches or representations to examine mathematical concepts so that students develop a better understanding of connections among topics and improve their ability to work abstractly Require group and individual explorations to promote the ability of students to communicate orally and in writing, including mathematical notation and terminology Use technology to promote student discovery, develop concepts, examine multiple perspectives, and give students experience with the technology skills that they will use in their careers Work with instructors in other disciplines to develop learning communities that pair a mathematics class with a class in another department Provide guest speakers from scientific fields for student mathematics clubs or for selected classes and encourage student participation in professional organizations through activities such as the AMATYC Student Math League, MAA student chapters, and Mu Alpha Theta mathematics organizations Recruit students, including those from underrepresented groups, into mathematics-intensive programs and careers

Standards in Course Design

The AMATYC IMPACT standards support recent work in the profession, including the Common Vision document, and add some additional frameworks for developing an effective and up-to-date course. This section deals with the macro-level choices made about courses, while the Learning Environment section deals with micro-level choices within the course.

Decisions about course design articulate how the curriculum is going to be delivered to students in ways that promote PROWESS: PRoficiency, OWnership, Engagement and Student Success. These decisions are best viewed as a joint responsibility by all faculty involved with a given course, including a joint decision on ranges of acceptable variation between sections and delivery methods.

The AMATYC IMPACT standards provide the following general goals for course design in the first two years of college mathematics:

- Effective instruction is not limited to a specific style of teaching; effective course design incorporates diverse styles within and across courses. Current work in the cognitive sciences should inform the instructional design to intentionally support students in achieving the identified student learning outcomes.
- Increasing and maintaining high levels of student engagement will contribute to building proficiency and student success.
- Providing support for students to develop a more diverse set of learning skills is more important than attempting to match students' "learning styles"; learning styles is not a stable cognitive construct, while "learning skills" has a sound research basis.
- Course materials might be traditional textbooks, e-books, or Open Educational Resources (OER); the selection of these materials should be based on criteria related to quality, effectiveness, and affordability.
- Learning technology should be included in all mathematics courses, and the use of technology should support the curricular goals and course outcomes. Mobile devices (both calculators and smart phones) need to be part of this mix of technology.
- Alternatives to the traditional semester or quarter length courses could enhance the mathematics experience for some cohorts of students. This might include co-requisite structures, fast track courses, and individualized learning.

Implied within the curriculum and course design standards is the goal of every math course contributing to helping students become better and more skilled learners. Every course design should include some attention to this goal, regardless of level of course or format of delivery.

The course design also needs to support the wide diversity of our students, both culturally and physically. (For this purpose, we include learning disabilities in the physical category of diversity.) Faculty must recognize that diversity manifests itself in a variety of ways: age, gender, ethnicity, socio-economic background, and academic preparation. Achieving PROWESS requires that colleges provide all students with both opportunities and assistance, including students with significant language differences. The course design should be based on our success goal … each and every student will pass a given course.

The chart below captures specific standards for categories in course design.

Diversity of Students	Equity for Students	Learning Skills
Have high expectations of all students and clearly communicate those expectations to students While the student population may be diverse, the educational outcomes for a course must be expected of all students. Be aware of and accommodate diverse student needs Collaborate with appropriate support services personnel to respond to the needs of students with disabilities Serve as student mentors and mathematics advisors Advise students of the availability and appropriate use of academic support resources Utilize specific teaching strategies for different populations of students/courses Focus on building student motivation	Use 'best practices' for increasing success rates for minority students Strive to include underrepresented demographic groups in mathematics Use multiple types of assessment measures Use strategies shown to reduce "stereotype threat" Broaden the foci of each course beyond procedural accuracy Emphasize communication as a process Use strategies shown to build a sense of inclusion and community	Be sensitive to the fact that many students are balancing family, job, and academic responsibilities; provide constructive suggestions and support in the process Provide explicit expectations concerning student effort and behavior (attendance, use of social media during class, homework completion, seeking assistance, etc). Explicitly direct students to consider their own cognitive processes; develop metacognitive skills
Face-to-Face group formats	**Online or Distance Learning**	**Individualized formats**
Incorporate multiple pedagogical approaches (lecture, whole class activity, small group work, and individual work) Design instruction and assessments so that individual students accept their responsibilities for learning mathematics, even if group processes are heavily used in the classroom	Select technology that is accessible to students enrolled in their distance learning mathematics course Assure that learning outcomes in mathematics distance learning sections are consistent with those of similar mathematics courses taught in classrooms	Maintain high levels of faculty-student interaction Select technology that is accessible to students enrolled in their distance learning mathematics course Assure that learning outcomes in mathematics distance learning sections are consistent with those of similar mathematics courses taught in classrooms

Learning environment

In all course designs, the learning environment is where students experience mathematics with the guidance of faculty. The details of a learning environment support the curriculum and course design so that students achieve PROWESS. The word "classroom" also refers to the learning environment, though we prefer the broader term "learning environment" to include all settings in which faculty and students interact with and learn mathematics.

The AMATYC IMPACT standards provide the following general goals for the learning environment in the first two years of college mathematics:

- The quality of the interactions in the learning environment determines the effectiveness in mastering mathematics.
- The learning environment should foster active student engagement in mathematical thinking, encourages student creativity and risk-taking, and promotes a culture that values the diverse interests and backgrounds of students.
- The learning environment should furnish students and faculty with the appropriate physical space, materials, technological resources, and support staff necessary to facilitate effective learning of mathematical concepts and skills.
- Faculty should use innovative teaching and learning strategies that incorporate technology and learning activities designed to promote active student engagement, meaningful discourse and cooperative learning.
- The learning environment should be designed to be effective in developing PROWESS, which includes increasing students' persistence, grit, and communication skills
- Other goals include: Respect diverse talents and ways of learning and teaching; use a variety of classroom activities, assignments, and assessments; encourage student-faculty contact; provide students with prompt feedback

The chart below captures specific standards for the learning environment.

Student Diversity	Factors that Influence Learning	Connect to Outside the Classroom
Have high expectations of all students and clearly communicate those expectations to students Be aware of and accommodate diverse student needs Collaborate with appropriate support services personnel to respond to the needs of students with disabilities Serve as student mentors and mathematics advisors Advise students of the availability and appropriate use of academic support resources Use 'best practices' for increasing success rates for minority students Strive to include underrepresented demographic groups in mathematics Utilize specific teaching strategies for different populations of students/courses Directly encourage student motivation for all students Be sensitive to the fact that many students are balancing family, job, and academic responsibilities; provide constructive suggestions and support in the process.	Make mathematics accessible to all students, being sensitive to the impact of mathematics anxiety on students and teach students to employ strategies to control, manage, and reduce student anxiety Provide students with course information outlining course objectives, concepts, ideas, and learning outcomes for their mathematics course Structure the learning environment so that each student is likely to both learn new material and to correct prior knowledge as needed, in order to create cognitive schema which are more stable and productive in future work Advise students on the expectations of their distance learning mathematics course and orient them to the distance learning environment for their course Be aware of the diverse mathematics backgrounds of their students Answer questions and explain material carefully and clearly, being patient, supportive, and available to help when students are frustrated or confused Regularly require students to work on mathematics outside the classroom. This will include expecting students to prepare for class as well as to practice what is done in class. Instructors will encourage these behaviors with timely feedback Use multiple categories of assessment measures, including items that differentiate situational, institutional, and dispositional factors. Use the history of mathematics to engage learning	Refer students to appropriate support services for help in reducing mathematics anxiety Be involved in the design of and the decision-making about physical spaces that support mathematics instruction Identify and recommend needed technology Encourage appropriate interaction with students and between students inside and outside of the classroom Be available outside of the classroom to assist individual students Provide service learning opportunities for students in your courses Foster undergraduate research.

Instructor Talk/Lecture	Group Work	Individual Work
Instructor talk should support student engagement with the material Lecturing should be focused on building meaningful mastery of the learning outcomes, as should all methods of instruction. Use questioning to promote student engagement and to measure student understanding	Structure for positive interdependence: Group interaction is necessary for successful resolution of the question or task; linking individual success and the success of the group [1] Structure face-to-face interaction: Group interactions include discussing solution paths, important concepts, and connections to prior knowledge and facilitating words of encouragement and help when needed [1] Structure individual accountability: Students are held accountable for their share of the work in the group [1] Structure social skills: Group interaction requires interpersonal, social, and collaborative skills. Students must be provided guidance in how to effectively interact in a small group [1] Structure group processing: Group members discuss effectiveness in reaching their goals and in working together [1] Faculty should receive training on the techniques and justification for the specific group work done (think-pair-share, paired-board work, jig-saw teaching, etc)	Individual work should occur frequently in mathematics classrooms Individual work should include all levels of learning (conceptual, procedural…), not merely exercises that mimic recently explained procedures. Technology is a valuable tool for individualizing learning, but not a replacement for interaction with mathematics faculty

[1] These items taken from Johnson and Johnson (1999)

Assessment and Maintenance

Assessment refers to processes that provide information on the nature and quality of learning. This feedback is critical for all three components discussed: Curriculum (called "Program" assessment here), Course Design, and Learning Environment (called "Classroom Assessment). A single assessment activity might produce information on just one component, or it might address multiple components.

In all cases, the purpose of assessment is to improve learning and build PROWESS.

The AMATYC IMPACT standards provide the following general goals for assessment in the first two years of college mathematics:

- Assessment is an ongoing process of collecting pertinent evidence that informs instructors about students' understandings of mathematical concepts, the student's proficiency with specific procedures, and the student's ability to apply various problem-solving strategies in solving both routine and non-routine problems.
- Assessment should incorporate authentic assessments, which strive to evaluate students' abilities in real-world contexts.
- Assessments should focus on analytical skills, the ability for students to integrate what they learn, have students work collaboratively, and articulate their thinking in written and oral modes.

Assessments are used at different levels … each classroom, each course, and each program. Each level seeks to provide both measurements against an acceptable level and measurements of improvement over time. The validity and reliability of assessments should be measured and developed as part of the process – presumptions about either validity or reliability of particular assessments need to be supported by evidence.

Classroom Assessment	Course Assessment	Program Assessment
Incorporate classroom assessment activities into the classroom learning environment on a regular basis; learning and assessment are concurrent processes Provide feedback at times and in ways that are most helpful to student learning Adjust classroom activities in response to assessment information Discuss assessment results with students and explain how the information is being used to make instructional decisions Use a variety of assessment techniques including formative, summative, and authentic assessments Use assessment data as a learning tool to address misconceptions and misunderstandings.	In conjunction with other faculty, locally and/or regionally, agree upon the core student learning outcomes for each mathematics course Communicate course outcomes to students at the beginning of the course. Use course assessments to measure achievement of those outcomes and determine needed improvements Use results of assessment to improve the learning environment during the course and in subsequent semesters	Identify assessment tools linked to desired student learning outcomes and proceed through the assessment implementation cycle to implement improvements Develop assessments to monitor placement and progression in sequences and pathways Participate in the development and assessment of general education outcomes in mathematics Determine which of the general education outcomes are met when students complete a given mathematics course Continually use assessment results to evaluate program effectiveness