Forty Five Years of Dev Math in 50 Minutes

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AMATYC 2017

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S137, Sat 11:55-12:45

Start with "Now"!

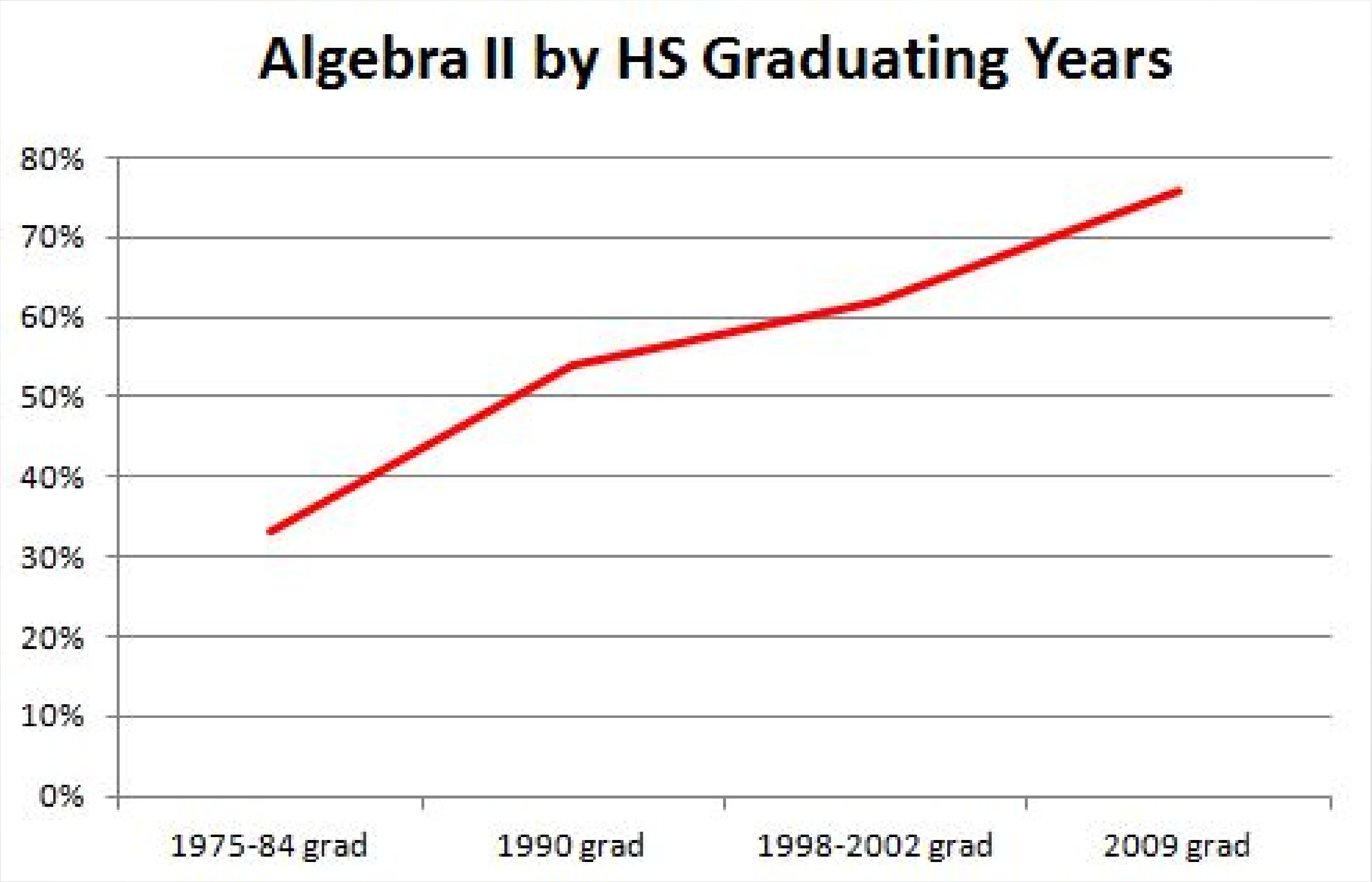
- Minimization: smaller footprint for developmental mathematics
- Trend A: Co-requisite remediation (footprint size=0)
- Trend B: Pathways (smaller footprint for sub-populations)
- Trend C: Replace traditional dev math with modern courses (smaller footprint for all)
- Everybody is an expert (even college presidents and system chancellors)

It's the Mathematics, silly!

In the beginning …

- Developmental mathematics … kinder, gentler remedial mathematics
- Complete the mathematics college-prep kids did in high school, for those who did not
- “High school” mathematics cloned
- Was anti “New Math” (in general)
- Rationale: Get students ready for College Algebra or equivalent

Made some sense then … (1975)



Bureau of Labor Statistics, "High school math courses and college attendance in two generations"
http://www.bls.gov/opub/ted/2012/ted_20121016.htm
NCES Fast Facts: Advanced Mathematics & Science https://nces.ed.gov/fastfacts/display.asp?id=97

The Dev Math Curriculum … 1975

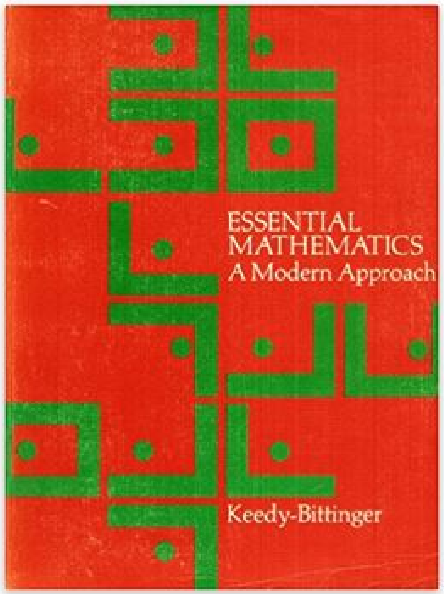
- Basic Math (<8th grade)
- Pre-Algebra (8th grade)
- Beginning Algebra (9th grade)
- Intermediate Algebra (10th or 11th grade)
- Few had Geometry (10th or 11th grade)

What we tried then (1975)

- Workbooks
- Programmed instruction books
  and specialized learning machines
- Audio tapes
- Books in 3 colors

- It was all about the **materials**

A sample …



Essential Mathematics: Modern Approach Paperback – Import, December, 1972

by Mervin L. Keedy (Author), Marvin L. Bittinger (Author)

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Hardcover from $50.00	**Paperback from $49.99**
2 Used from $50.00	1 Used from $49.99

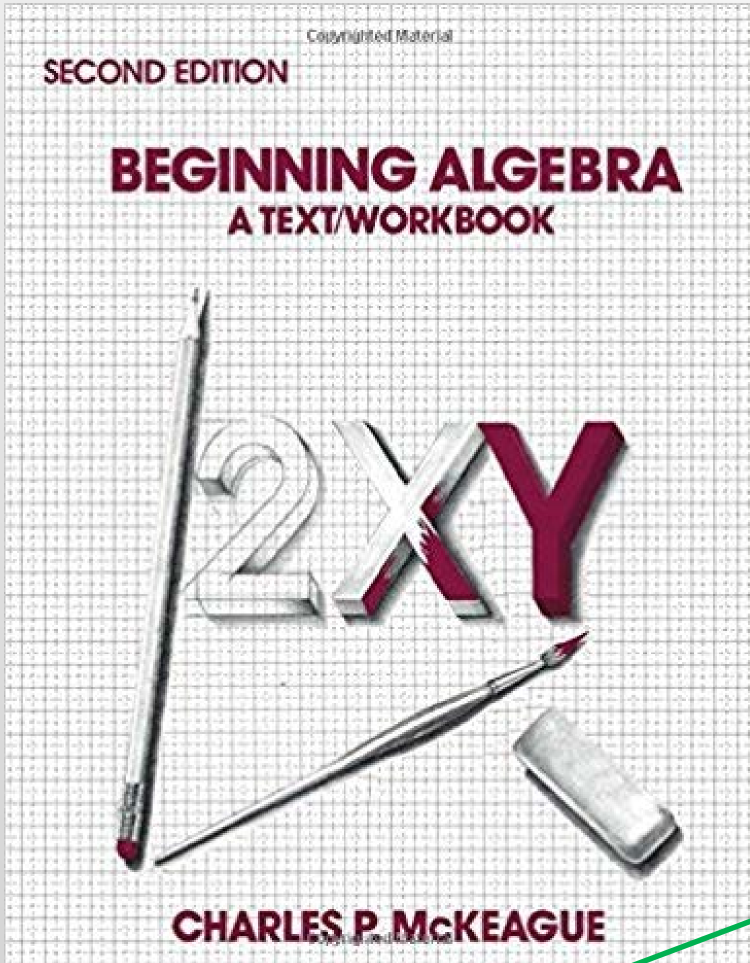
This book contains a treatment of arithmetic, algebra, and trigonometry, and an appendix on the slide rule. The text adapts to either review or first time coverage and is designed mainly for a junior college audience with no prerequisites necessary. Practical application is emphasized throughout, with a minimum of words used to explain concepts; this stress on demonstration over description is especially helpful for remedial students. Of particular interest is the textbook page with wide margins intended for exercises a feature that creates an opportunity for student reinforcement and eliminates the cost of an extra workbook.

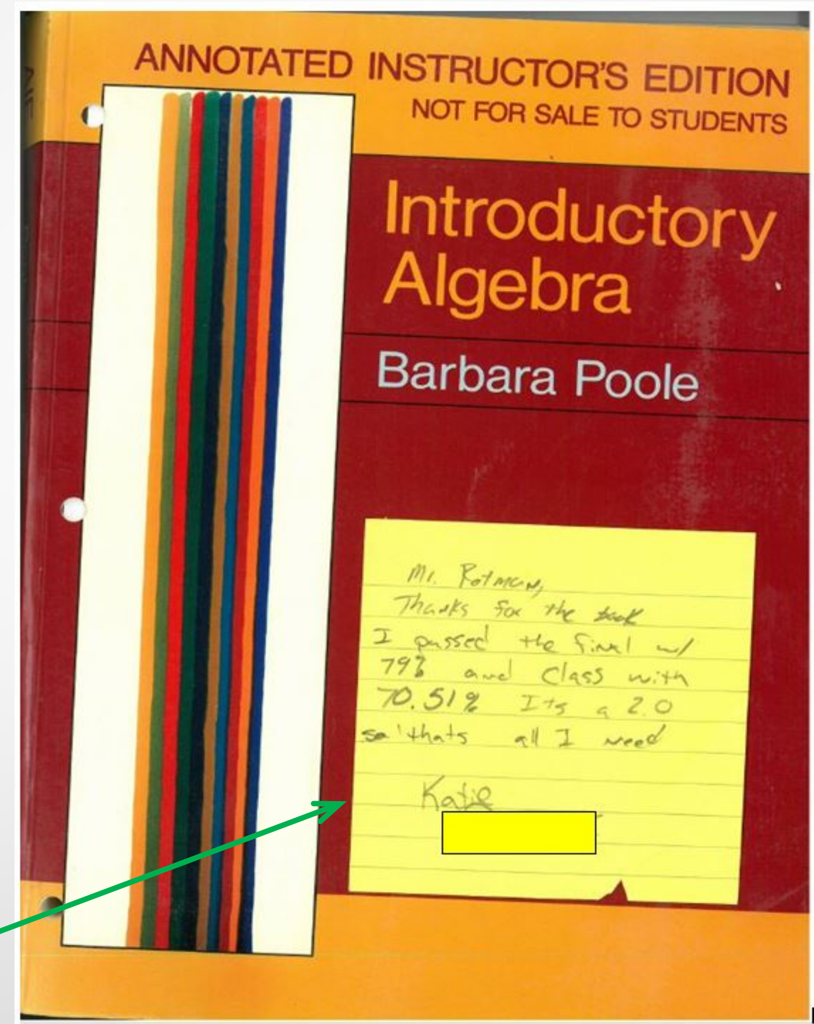
“Slide Rule” was the computing device of the era. “Minimum of words” was a goal in many textbooks of the day.

Decade 2: the 1980's

- Arithmetic skills obsession (reaction to 'handheld calculators')
- Low pass rates meant "let's add another course!"
- Student Learning Problems (aka "blame the student")
- Back to Basics (skills, procedures)

Samples from the 1980's:





"Mr. Rotman: Thanks for the book. I passed the final w/79% and class with 70.51%. It's a 2.0 so that's all I need".

A Typical Section …

5.3 MULTIPLICATION OF POLYNOMIALS

1 Multiplying Monomials

Recall from Section 2.7 that when we multiply two exponential numbers with the same base, we keep the base and add the exponents.

$$a^m \cdot b^m = a^{m+n}$$

> To multiply two monomials multiply the coefficients and add the exponents on the identical variables.

EXAMPLE 1 Multiply.

a. $5(4x^2) = 5(4)(x^2) = 20x^2$

b. $3x(5x) = (3)(5)(x)(x) = 15x^2$

c. $(-6x^2y)(2x^3z^3) = (-6)(2)(x^2y)(x^3z^3) = -12x^5yz^3$

d. $-6ab(4a^2b^3) = (-6)(4)(ab)(a^2b^3) = -24a^3b^4$ Recall $ab = a^1b^1$ ■

□ DO EXERCISE 1.

2 Multiplying a Monomial and Any Polynomial

To multiply a monomial times any polynomial, multiply each term of the polynomial by the monomial. This is using a distributive law. Sometimes we apply it to more than two terms.

EXAMPLE 2 Multiply.

a. $4x(x^2 + 3) = 4x(x^2) + 4x(3)$ Using a distributive law
$= 4x^3 + 12x$

b. $-2y(y^2 + 4y - 7)$
$= -2y(y^2) + (-2y)(4y) - (-2y)(7)$ Distributive law for three terms
$= -2y^3 - 8y^2 + 14y$

Recall that a distributive law may be stated as $(b + c)a = ba + ca$.

c. $(x^2 - 2x + 3)3x = x^2(3x) - 2x(3x) + 3(3x) = 3x^3 - 6x^2 + 9x$ ■

□ DO EXERCISE 2.

OBJECTIVES

1. *Multiply monomials*
2. *Multiply a monomial and any polynomial*
3. *Multiply two polynomials*
4. *Multiply a binomial by a binomial using FOIL*

□ **Exercise 1** Multiply.

a. $-7(3y)$
$-21y$

b. $-x(4x^2)$
$-4x^3$

c. $8xy^2(-2x^3y^5)$
$-16x^4y^7$

d. $-x(-8x^6y^4)$
$8x^7y^4$

□ **Exercise 2** Multiply.

a. $-2x^2(x + 4)$
$-2x^3 - 8x^2$

b. $3y(2y^2 - 4y + 8)$
$6y^3 - 12y^2 + 24y$

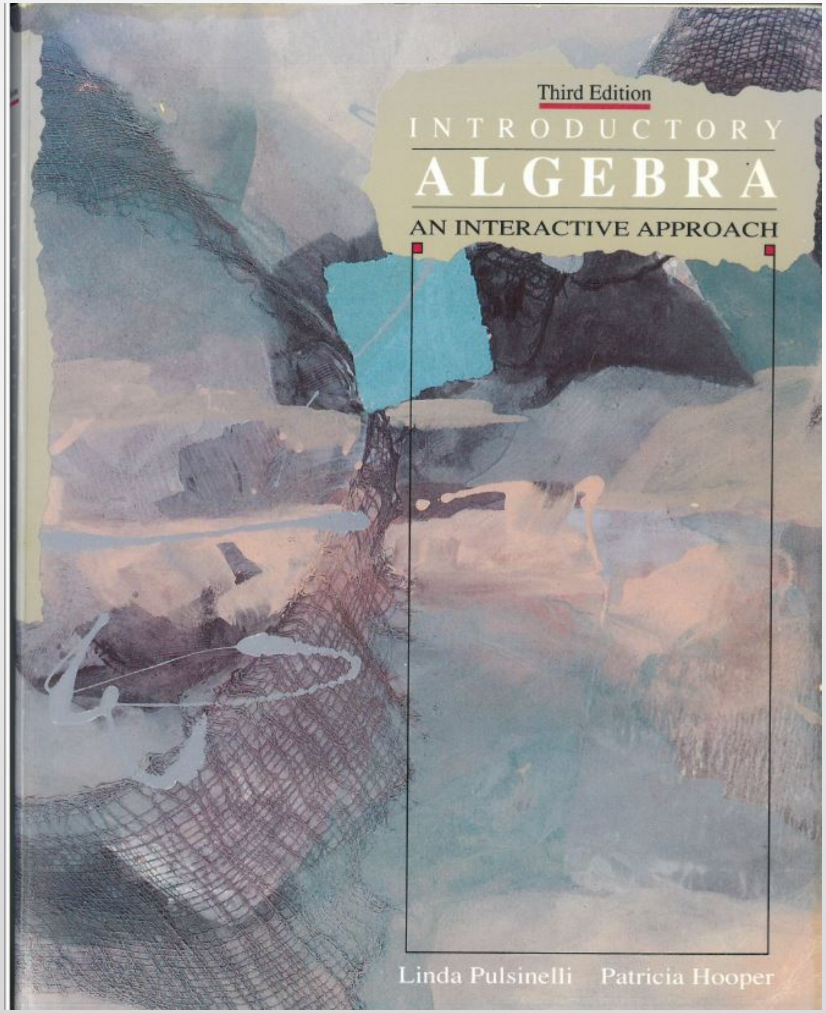
c. $5y(y^3 - 7)$
$5y^4 - 35y$

d. $-x(-x^2 + 3x - 1)$

Decade 3: Early 1990s

- NCTM Standards … small changes for us
- Graphing calculators … all or nothing [Most of us did 'nothing']
- "Time for a change" (Ed Laughbaum)
- Many of the same messages then … as in Common Vision & Math Sciences 2025
- We still focused on: old curriculum, getting students ready for College Algebra

Sample from early 1990s



A Typical Section …

6.2 Multiplying with Monomials

Multiplying Monomials

In Chapter 5 we learned to use the First Law of Exponents to multiply powers of the same base. Recall that $a^m \cdot a^n = a^{m+n}$. We use the commutative and associative properties of multiplication when multiplying monomials containing constants *and* variables.

Example 1. Find $2x^3(4x^2)$.

Solution

$$2x^3(4x^2) = 2 \cdot x^3 \cdot 4 \cdot x^2$$
$$= 2 \cdot 4 \cdot x^3 \cdot x^2$$
$$= 8 \cdot x^{3+2}$$
$$= 8x^5$$

You try to complete Example 2.

Example 2. Find $-5y^8(6y^7)$.

Solution

$$-5y^8(6y^7) = -5 \cdot y^8 \cdot 6 \cdot y^7$$
$$= -5 \cdot 6 \cdot y^8 \cdot y^7$$

Check your work on page 281. ▶

To multiply monomials, first multiply the numerical coefficients, then multiply the variables using the First Law of Exponents.

Example 3. Find $4a^5\left(\frac{1}{3}a^7\right)$.

Solution

$$4a^5\left(\frac{1}{3}a^7\right) = 4 \cdot \frac{1}{3} \cdot a^5 \cdot a^7$$
$$= \frac{4}{3} \cdot a^{5+7}$$
$$= \frac{4}{3}a^{12}$$

You complete Example 4.

Example 4. Find $3x^2(-2x^4)(-7x)$.

Solution

$$3x^2(-2x^4)(-7x)$$
$$= 3(-2)(-7) \cdot x^2 \cdot x^4 \cdot x$$

Check your work on page 281. ▶

Example 5. Find $0.8x(3x^2y)$.

Solution

$$0.8x(3x^2y)$$
$$= 0.8 \cdot 3 \cdot x \cdot x^2 \cdot y$$
$$= 2.4x^3y$$

Try to complete Example 6.

Example 6. Find $(-2xy^2z)(-9x^2y^5z^7)$.

Solution

$$(-2xy^2z)(-9x^2y^5z^7)$$
$$= (-2)(-9) \cdot x \cdot x^2 \cdot y^2 \cdot y^5 \cdot z \cdot z^7$$
$$= \_\_\_\_\_\_$$

Check your work on page 282. ▶

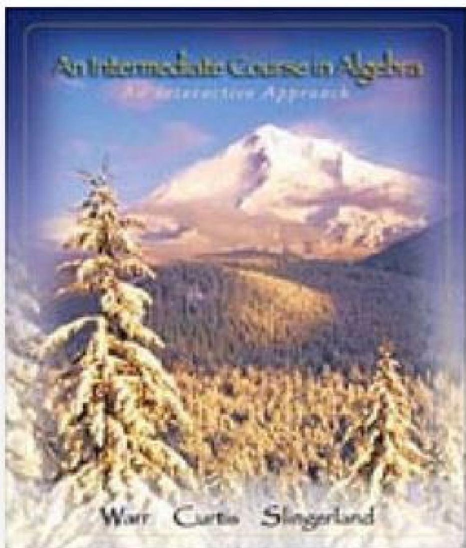
Decade 3: Late 1990s

- Pockets of reform and revolution:
  Focus on writing textbook(s); some grant based
- Supported by AMATYC Standards (1995) and NCTM standards (though not by 'us')
- Presentations at AMATYC and affiliates
- Some were similar to current "Option C"
  Replace traditional dev math with modern courses

One of the 1990s Reform Books

An Intermediate Course in Algebra: An Interactive Approach (with InfoTrac) / Edition 1

by Alison Warr, Cathy Curtis, Penny Slingerland



ISBN-10: 0534436730

Hardcover

$12.64

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Seller Comments:

0534436730 Item in good condition. Textbooks may not include supplemental items i.e. CDs, access codes etc... All day low prices, buy fror us sell to us we do it all!!

"This book was written to address the challenge of the NCTM and AMATYC Standards and technology integration in the classroom. The authors address the standards using a variety of methods, including Numerical, Graphical, and Algebraic Models; Guided Discovery Activities; Problem Solving; Technology; Collaborative Learning."

Another sample (1990s reform)

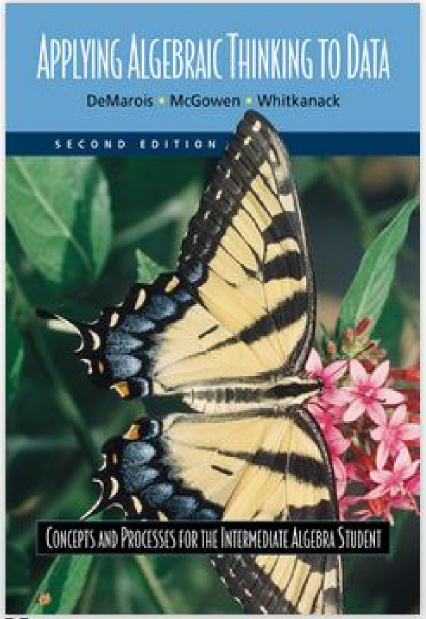
Applying Algebraic Thinking to Data: Concepts and Processes for the Intermediate Algebra Student, 2nd Edition

Phil DeMarois, Mt. Hood Community College

Mercedes McGowen, William Rainey Harper College

Darlene Whitkanack, University of Illinois, Chicago

©2001 | Pearson | Out of print



View Larger

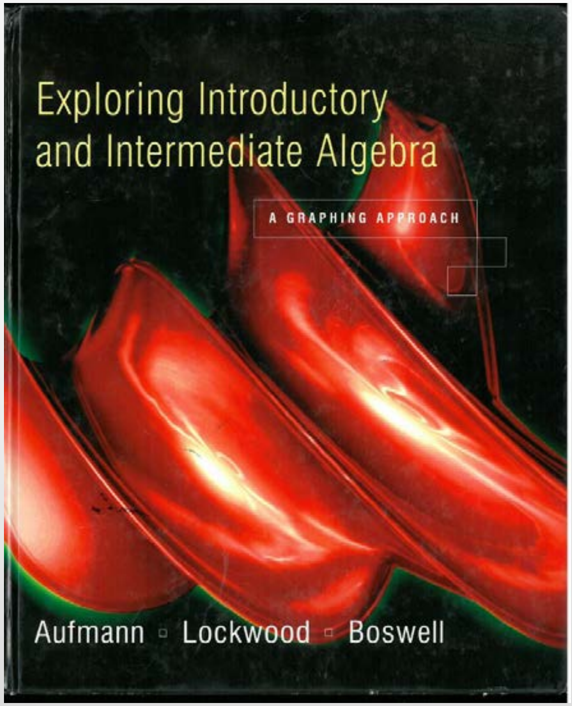
Did those reform books do any good?

- The books themselves served ‘niche markets’
- However: Some of the current reform materials are similar (i.e., Mathematical Literacy … Algebraic Literacy)

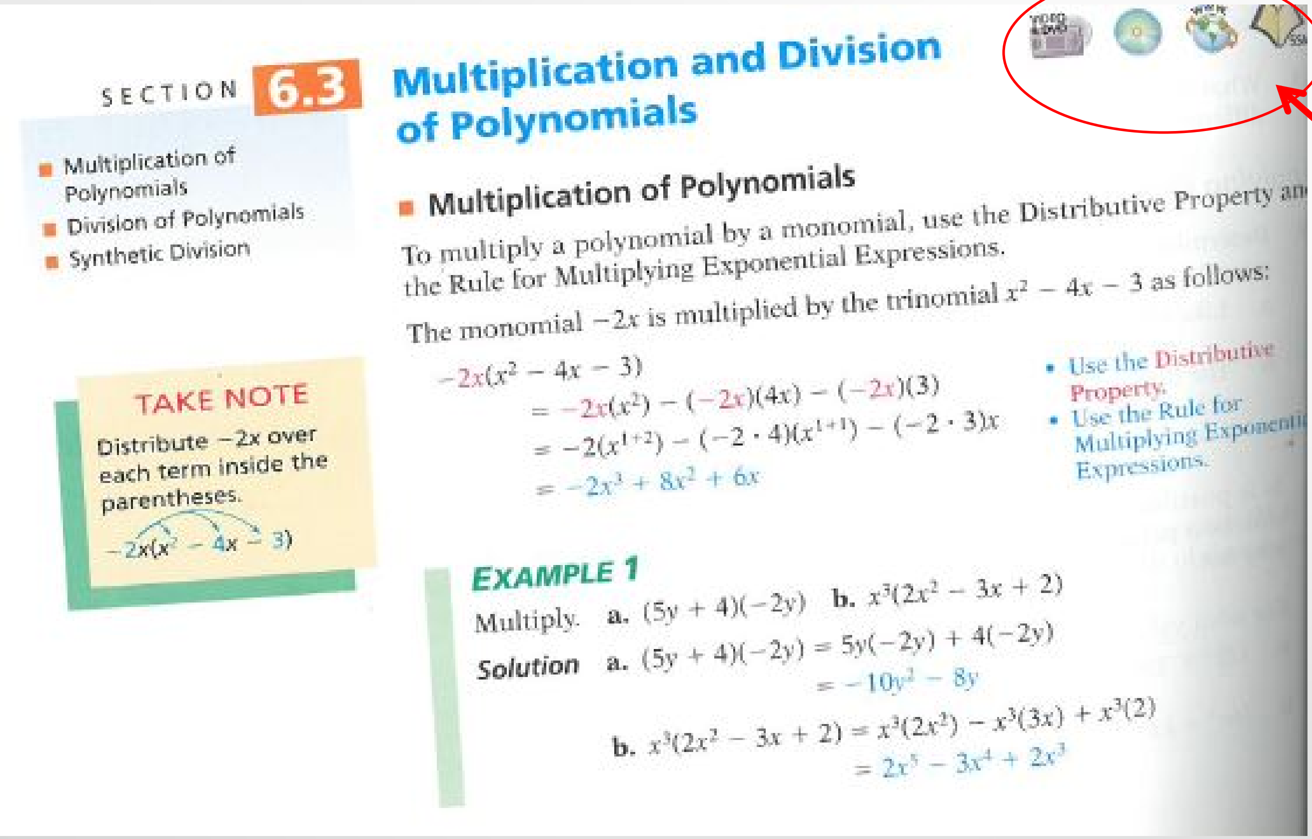
Decade 4: 2000 to 2009

- Publisher's Golden Age: lots happening
- Digital as supplement
- Focus on commonly used content
- Reduction in reform books, and birth of combined algebra texts
- Separate and unequal: graphing calculator within some textbooks; most avoid GC
- Few of us thought of anything besides College Algebra

Text sample … 2004



A Typical Section



SECTION 6.3

- Multiplication of Polynomials
- Division of Polynomials
- Synthetic Division

Multiplication and Division of Polynomials

Multiplication of Polynomials

To multiply a polynomial by a monomial, use the Distributive Property an[d] the Rule for Multiplying Exponential Expressions.

The monomial $-2x$ is multiplied by the trinomial $x^2 - 4x - 3$ as follows:

$$-2x(x^2 - 4x - 3)$$
$$= -2x(x^2) - (-2x)(4x) - (-2x)(3)$$
$$= -2(x^{1+2}) - (-2 \cdot 4)(x^{1+1}) - (-2 \cdot 3)x$$
$$= -2x^3 + 8x^2 + 6x$$

- Use the Distributive Property.
- Use the Rule for Multiplying Exponenti[al] Expressions.

TAKE NOTE

Distribute $-2x$ over each term inside the parentheses.

$-2x(x^2 - 4x - 3)$

EXAMPLE 1

Multiply. **a.** $(5y + 4)(-2y)$ **b.** $x^3(2x^2 - 3x + 2)$

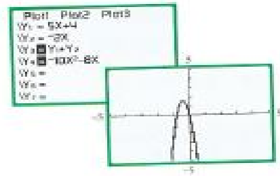
Solution **a.** $(5y + 4)(-2y) = 5y(-2y) + 4(-2y)$
$= -10y^2 - 8y$

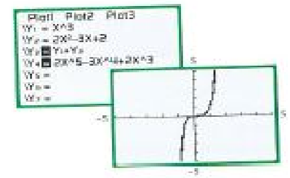
b. $x^3(2x^2 - 3x + 2) = x^3(2x^2) - x^3(3x) + x^3(2)$
$= 2x^5 - 3x^4 + 2x^3$

A Typical Section

ALCULATOR NOTE
the graphs at the right
duced, the graph of Y3
top of the graph of Y4.
eans that the two
s are the same. It does
sure that the product is
t; however, if the two
s do not match exactly,
ultiplication is definitely
ect.

Check:





YOU TRY IT 1

Multiply. **a.** $(-2d + 3)(-4d)$ **b.** $-a^3(3a^2 + 2a - 7)$

Solution See page S24.

Multiplication of two polynomials requires the repeated application of the Distributive Property.

Shown below is the binomial $y - 2$ multiplied by the trinomial $y^2 + 3y + 1$.

$(y - 2)(y^2 + 3y + 1)$

$= (y - 2)(y^2) + (y - 2)(3y) + (y - 2)(1)$ • Use the Distributive Property to multiply $y - 2$ times each term of the trinomial.

$= y^3 - 2y^2 + 3y^2 - 6y + y - 2$ • Use the Distributive Property.

$= y^3 + y^2 - 5y - 2$ • Combine like terms.

Two polynomials can also be multiplied using a vertical format similar to that used for multiplication of whole numbers. Note that the factors in the multiplication below are the same as those used in the previous example.

$$\begin{array}{r} y^2 + 3y + 1 \\ y - 2 \\ \hline -2y^2 - 6y - 2 \\ y^3 + 3y^2 + y \qquad \\ \hline y^3 + y^2 - 5y - 2 \end{array}$$

• Multiply each term in the trinomial by -2.
• Multiply each term in the trinomial by y. Like terms must be written in the same column.
• Add the terms in each column.

EXAMPLE 2

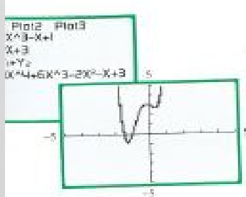
Multiply: $(2b^3 - b + 1)(2b + 3)$

Solution

$$\begin{array}{r} 2b^3 - b + 1 \\ 2b + 3 \\ \hline 6b^3 \qquad - 3b + 3 \\ 4b^4 \qquad - 2b^2 + 2b \qquad \\ \hline 4b^4 + 6b^3 - 2b^2 - b + 3 \end{array}$$

• This is $3(2b^3 - b + 1)$.
• This is $2b(2b^3 - b + 1)$. Like terms are in the same column.
• Add the terms in each column.

Check: A graphical check is shown at the left.



Decade 4: AMATYC Standards, Act 2

- Beyond Crossroads (2006)
- Process as a Focus ("Improvement Cycle")
- Curriculum addressed more in 1995 document
- Implicit acceptance of status quo (the out-of-date remediation structure)
- Policy influencers … began to be interested in developmental mathematics

Decade 4 (2000-2009): NCAT

- The Center for Academic Transformation
- Course Redesign as the all-purpose solution: Emporium; Modules
- Skills … old content
- Efficiency
- Generality: Isolated from the work of the profession

It's still the Mathematics, silly!

How Many People Does it Take?

- Three people might be enough … to start (2007)
- Fifteen people can create a new future for dev math (2009)
- Change and reform can grow when continuity exists in the profession
- Appeal to core beliefs of professionals: “Good mathematics” for all students

Decade 5: The Role of 2010

- Carnegie Foundation: Quantway™ and Statway™
- Dana Center: Foundations of Mathematical Reasoning
- AMATYC New Life: Mathematical Literacy, and Algebraic Literacy (the forgotten sibling)
- The "joyful conspiracy" (Uri Treisman)
- We began thinking about other college math courses (besides 'college algebra')

Decade 5: No Longer Hidden

- Prior to 2010, dev math operated under the radar
- Until … Policy influencers painted a dismal picture of our work
- Policy influencers sought to disrupt the continuity in the profession
- Specific solutions "sold" to college and system leaders (presidents, provosts)
- Focus on non- (or anti-) College Algebra

Minimization Option A: Footprint=0

- Co-requisite remediation as the all-purpose solution
- Focus on Statistics & Liberal Arts Math (or QR)
- "The data is in … co-requisite remediation works"
- "We can't a group of students for which it does not work." If it sounds too good to be true … is it?
- College algebra de-valued; get done with math!
- Would it pass the 'employment standard'?

Minimization Option B: Some Gain

- Pathways
- Students needing statistics or quantitative reasoning (aka “non-STEM”)
  arithmetic courses often still required;
  replaces 1 or 2 algebra courses (conditionally)
- “STEM” students generally see the same old curriculum (obsolete stuff) **The “Jekyll-Hyde” approach**
- Get students done with math but in programs which may have low employment rates

Minimization Option C: Replacement

- Mathematical needs relatively same for all students (at the Math Literacy level)
- Eliminate arithmetic (and pre-algebra)
- Algebra II is no longer sufficient for pre-calculus prep: Need Algebraic Literacy
- Supports College Algebra as well as ‘other mathematics’ (stat, QR, etc)
- Supports upward mobility (mid- and high-skill technical programs)

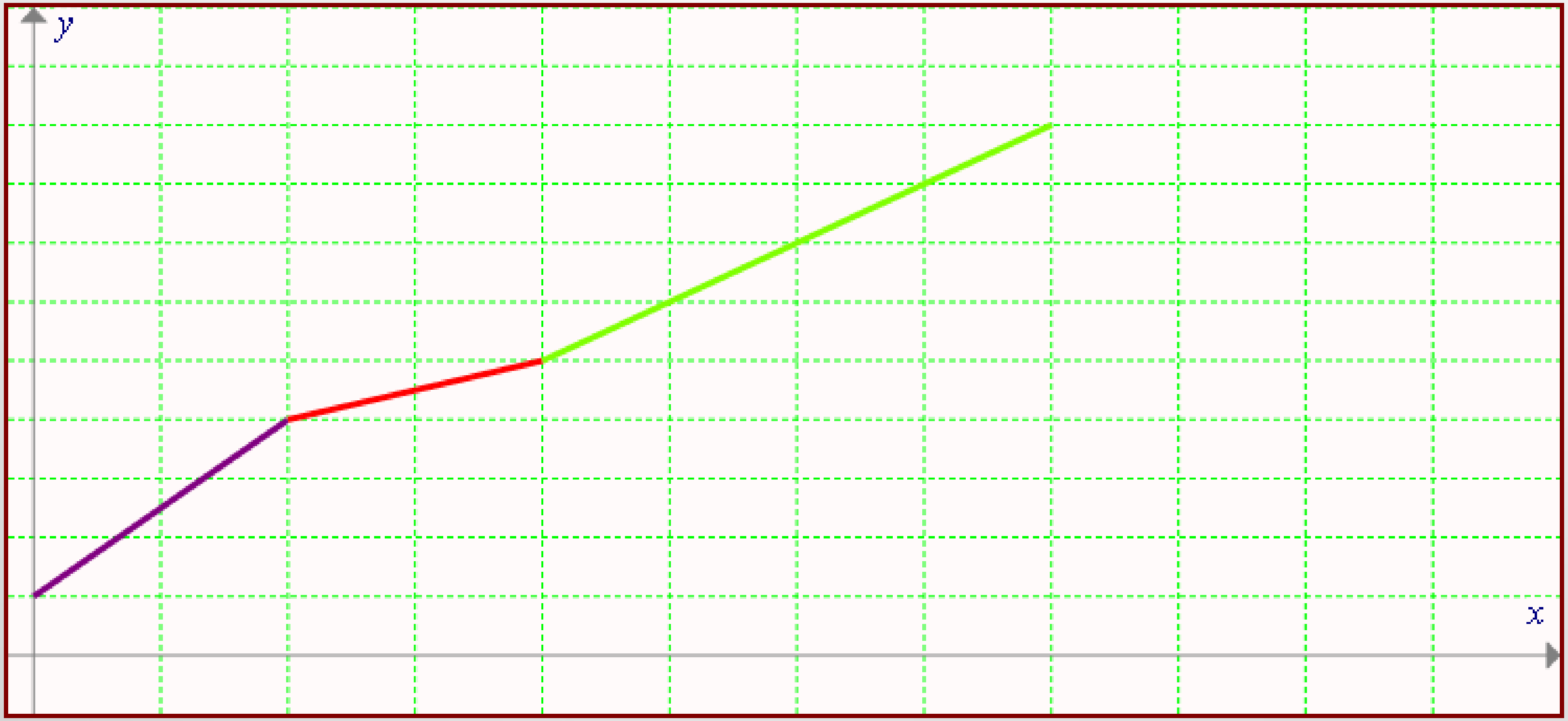
Is there an Option D?

- Not anytime soon
- Traditional Dev Math courses will not survive (perhaps 5 years)
- Policy influencers will not let us 'not change'
- College-Level courses will also shift to modern content … increasing the forces on dev math

The College Mathematics Curricula

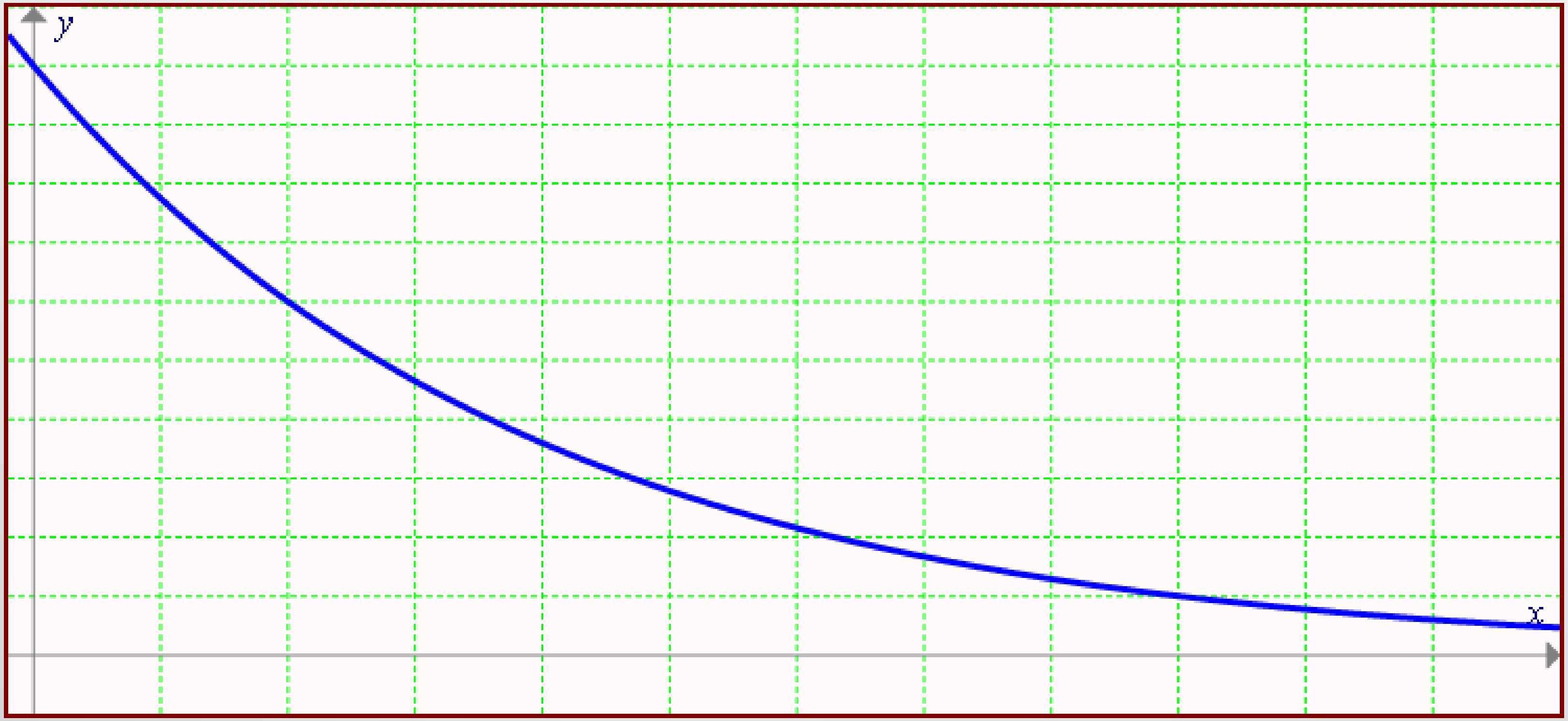
- Minimization also applies to college level math courses
- Obsolete content: will become modern, efficient
- Continuity is critical … our values, our dreams for ‘better’
- “Replacement” (option C) is our first step towards improving ALL of our courses

What WE see



"More developmental courses leads to more students being 'ready'!"

What THEY see

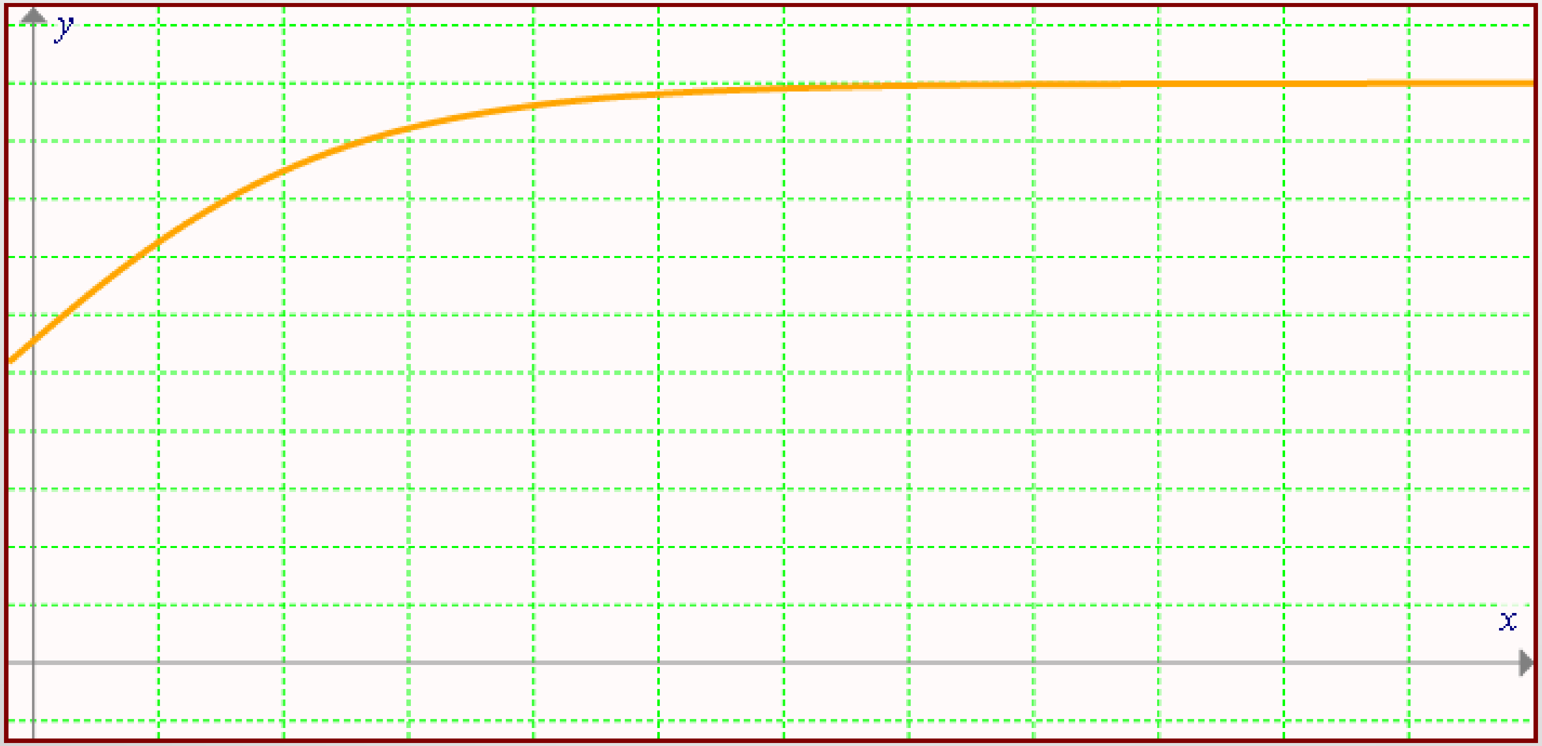


“More developmental courses means most students are blocked from degrees!”

“Remediation” will not survive

- Exponential decay is stronger: we can not WIN this argument
- Stop using the labels “remedial” and “developmental”
- Articulate a positive message about effective & modern preparation courses that we can show lead to success in ALL fields (not just non-STEM)
- Such as: **One (at most) pre-college prep course for 90% of students**

Our Future



"One course gets 90% of students ready for success in college!"

Remaining Challenges

- Do we accept the premise: “changing WHAT we teach is necessary for changing HOW we teach”?
- Can we articulate THE function? (And, is it an increasing or decreasing function?)
- Will we define the constraints? Or someone else?
- Among those constraints: needs of College Algebra, science courses (all of them) … and even statistics and QR

Where are we headed?

- All traditional developmental math courses will be gone within 5 years; several forces ensure that
- Survival of stand-alone "dev math" (prep) courses depends upon our professional work
- Co-requisite remediation will be an accepted solution; we must help define 'when' to use it
- Intro college math courses (up to Calculus ***n***) are the next field of dreams; who wants to play??

- **It's still about the mathematics!!**

Into the sunset …

- I could not ask for a better experience than I've had within AMATYC for these 30 years
- Any success I've had is based on the collaboration with other AMATYC members
- Each of us has a leadership role
- What will your role be?