# Lesson 4.x – Rates of Change and Health

We've seen situations where there is a constant 'adding' rate of change, and we know that this type of change is related to the general linear model $y = mx + b$. In our work with this model, the rate of change has been called the slope of the line. The rate of change is the same everyplace on the line. [If the rate of change varied, the graph would not be a straight line!]

**Additive change means a linear equation $y = mx + b$**
**The slope is the rate of change**

*Reminder: Rate of change is "output changes this amount per 1 unit of input change".*

We've also looked at situations where change is based on multiplying, and the exponential model. This lesson involves exponential patterns … and others.

Disease specialists use both kinds of information (per person, and per day). One tells us how contagious the disease is (number infected per person); the other estimates the number infected over time.

## Part A:

Infection diseases are tracked by specialists who are interested in predicting how many people will become ill. One recent infection had two characteristics: Each infected person passed along the infection to two other people, and the number infected grew by 5% each day.

First, here is a chart of the number infected starting with one person. The 'step' listed is not a time value like a day; the step represents people passing along the infection … which can happen several times in one day or could take several days.

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Number infected | 1 | 3 | 9 | 27 | 81 | 243 | 729 | 2187 |

*The danger of a disease spreading is measured by "R0" (two in this example). For more information, see http://ocw.jhsph.edu/courses/publichealthbiology/PDFs/Lecture2.pdf*

For the percent change table below, we start with 729 infected people on day 1.

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Number infected | 729 | 765 | 804 | 886 | 930 | 977 | 1026 | 1077 |

The equation for the second table is $D = 729(1 + 0.05)^x$ where x is the day number.

Questions:

1. Is either of these patterns linear (additive)?
*No, neither one involves a constant adding or subtracting.*

2. Is either of these patterns exponential (multiplicative)?
*Yes, both involve a repeated multiplying process.*

3. Estimate the rate of change on day 2 using data in the second table (measured by number infected per day).
$\frac{804-765}{3-2} = \frac{39}{1}$; *39 per day.*

4. Estimate the rate of change on day 7 using data in the second table (measured by number infected per day).
$\frac{1077-1026}{8-7} = \frac{51}{1}$; *51 per day.*



5. Is this rate of change constant or changing?
*The rate of change in changing (increasing each day).*

6. Estimate the rate of change on day 2 using data in the second table (measured by a percent increase).
$\frac{39}{765}$ *is about 5% increase per day.*

7. Does this percent increase relate to the percent increase in the description?
*Yes, it's the same 5%.*

**Quick Check 1**: If the balance is growing by 3% per year, what is the rate of change in year 50 (as a percent)?

Exponential functions can be stated in different forms. Scientific uses often involve the base *e* that we talked about before. We are using $y = A(1+r)^x$ as we focus on the rate of change.

**Percent change means a multiplying process.**
**The equation $y = A(1+r)^x$ fits the situation: A is the starting value.**
**The rate of change is r, the percent change (positive or negative).**

Some situations involve a percent decrease; a 4% percent decrease is a negative 4% rate of change … the multiplier is 0.96.

If we know the rate of change for the entire problem, we know the rate of change for every step within the problem. In cases where we do not have the change stated (either adding or multiplying), we can estimate the rate of change by using the data for particular input values.

## Part B:

Sometimes, a company will use a function to estimate the profit based on the number of units sold. For example: $P(n) = -.05x^2 + 16.5x - 100$ might give the profit P based on n, the number of units sold.

Questions:

8. Complete the table of values for the following inputs

| Units | 0 | 1 | 100 | 101 | 300 | 301 |
|---|---|---|---|---|---|---|
| Number infected | | | | | | |

*-100; -83.55; 1050; 1056.45; 350; 336.45*

On a graph, the rate of change at an input value is sometimes called the "tangent". This tangent is related to the tangent function for angles.

9. Use the data for 0 and 1 units to estimate the rate of change at 0 units.
$\frac{-83.55-(-100)}{1-0} = \frac{16.45}{1}$ *$16.45 per unit increase*

10. Use the data for 100 and 101 units to estimate the rate of change at 100 units.
$\frac{1056-1050}{101-100} = \frac{6}{1}$; *$6 per unit increase*

11. Use the data for 300 and 301 units to estimate the rate of change at 300 units.
$\frac{336.45-350}{301-300} = -\frac{13.55}{1}$; *-$13.55 per unit (a decrease)*

12. If the company has been making 100 units, would it help for them to increase production to more than 100 units?

*Since the rate of change is positive, yes … it will help some.*

13. If the company has been making 300 units, would it help for them to increase production to more than 300 units?

*No; the rate of change is negative … more units past 300 means less profit.*

**Quick Check 2**: Let $f(x)=3x-0.25x^2$. Use the function values for x = 20 and x = 21 to estimate the rate of change at x = 20. [Do not use percents.]

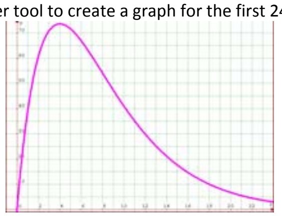
## Part C:

When we take a medication, there are various patterns that the drug can take in terms of how much is active in our body. One of the most common patterns involves a special number *e*; your calculator has a separate key for *e*, usually shown as $e^x$. For one drug, a dose has a pattern shown by $B(t)=t\cdot 50\cdot e^{-0.25t}$ where B is in milligrams, and t is the number of hours after the dose is administered.

The area of mathematics that studies rates of change in detail is called calculus. The problems that we are doing from data values would be done using symbolic methods in calculus. The tools of calculus also enable us to find the function if we know the rate of change for a variety of patterns.

Questions:

14. Use your graphing calculator or other tool to create a graph for the first 24 hours.



15. Use the data for hours 2 and 3 to estimate the rate of change at hour 2.

$\frac{70.9-60.7}{3-2}=\frac{10.2}{1}$ *; 10.2 milligrams per hour increase*

16. Use the data for hours 6 and 7 to estimate the rate of change at hour 6.

$\frac{60.8-66.9}{7-6}=-\frac{6.1}{1}$ *; 6.1 milligrams per hour decrease (negative rate of change)*

Our method lets use estimate a rate of change for an input value. Calculus uses a tool called a 'derivative' to provide an exact value for the rate of change at an input value.

17. Estimate the time (hours after the dose) when the rate of change is zero … neither increasing nor decreasing based on the graph.

*At hour 4*

18. Does this medication increase faster at the start or decrease faster towards the end?

*The start is 'faster'; the graph is steeper before hour 4 than it is after hour 4.*

19. Use the data for hours 6 and 6.1 to estimate the rate of change at hour 6.

$\frac{66.4-66.9}{6.1-6}=-\frac{.5}{.1}=-5$ *; 5 milligrams per hour decrease*

If needed, we can get a more precise rate of change by using input values closer together. Using 6 and 6.1 is more precise than using 6 and 7.

**Quick Check 3**: Let $P(t)=2000\cdot e^{0.1t}$. Use the function values for t = 10 and t = 11 to estimate the rate of change at t = 10. Write this in both forms (percent and not percent). Then use function values for t = 10 and t=10.1 to estimate the rate of change at t = 10.





## Part D:

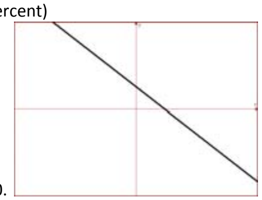
We can find basic 'rate of change' information from a graph, related to whether the rate of change is a constant addition or subtraction, or constant percent increase or decrease, or some other pattern.

**Rate of change:**
**Constant addition or subtraction ➔ Linear model**
**Constant percent increase or decrease ➔ Exponential model**

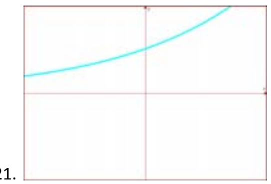
In general, an addition model is linear … a multiplying model is exponential.

Once we understand the overall rate of change information from a graph, we can classify the function as linear, exponential, or other.

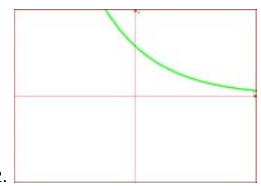
Questions: For each graph, classify the change (constant add or subtract, constant percent)

20. 

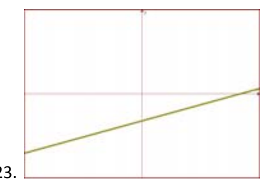
*Constant subtraction: LINEAR*

21. 

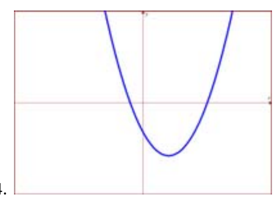
*Constant percent increase: EXPONENTIAL*

22. 

*Constant percent decrease: EXPONENTIAL*

23. 

*Constant addition: LINEAR*

24. 

*Neither: Not linear nor exponential*

If the rate of change is neither linear nor exponential, we will not be expected to identify the type. (This particular function is quadratic, something like $y=x^2-5x-6$.)