A Bridge to Somewhere: Algebraic Literacy

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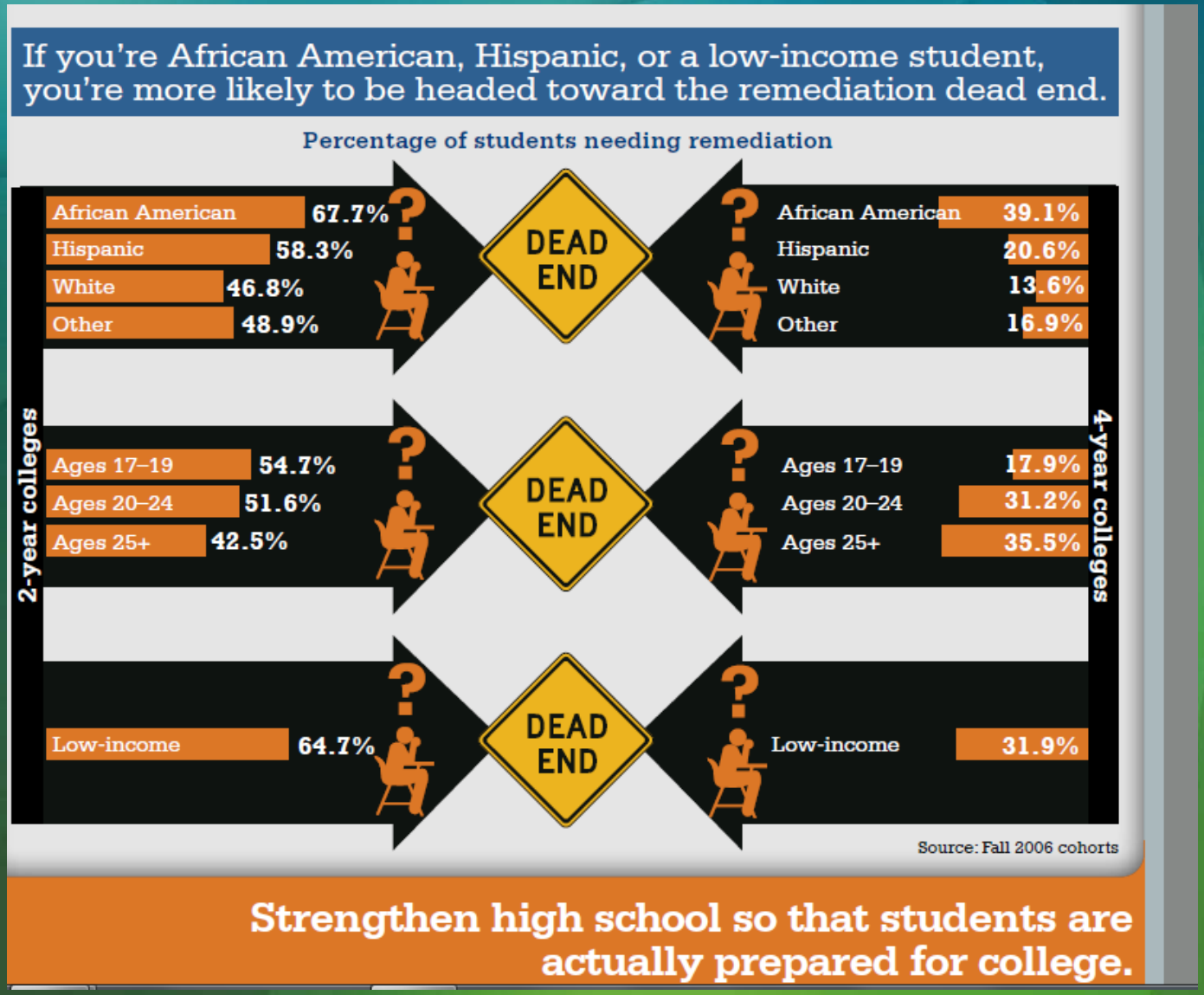
Outline

- “Bridge To Nowhere” report; other data
- Intermediate Algebra as a relic
- Engineering the Content (“backwards”)
- Algebraic Literacy course content
- Sample lessons from Algebraic Literacy

Bridge to Nowhere (sample page)



The "Nowhere" Data

REMEDIAL EDUCATION
Mathematics and English Success in 2-Year Colleges

	Hispanic			African American, non-Hispanic			White, non-Hispanic			Other		
	% enrolling in remedial courses	% remedial enrollers completing remedial courses	% remedial enrollers completing remedial and college-level courses	% enrolling in remedial courses	% remedial enrollers completing remedial courses	% remedial enrollers completing remedial and college-level courses	% enrolling in remedial courses	% remedial enrollers completing remedial courses	% remedial enrollers completing remedial and college-level courses	% enrolling in remedial courses	% remedial enrollers completing remedial courses	% remedial enrollers completing remedial and college-level courses
Arizona	30.7%	15.2%	7.7%	25.9%	9.6%	5.2%	12.1%	12.3%	5.8%	24.7%	13.4%	6.2%
Arkansas	29.2%	61.5%	37.2%	38.2%	41.7%	25.5%	22.9%	54.9%	35.4%	25.7%	57.4%	36.1%
California (CSU system only)	NP	NP	NP	NP	NP	NP	NP	NP	NP	NP	NP	NP
Colorado	19.4%	42.4%	49.6%	23.8%	34.5%	36.5%	9.7%	50.8%	67.1%	10.9%	46.9%	60.4%
Florida	23.6%	51.4%	12.9%	38.1%	45.5%	10.3%	17.9%	50.4%	12.4%	19.9%	58.3%	16.9%
Georgia	10.5%	45.8%	8.4%	16.2%	37.6%	7.8%	6.3%	52.1%	15.8%	9.9%	42.4%	20.1%
Hawaii	22.2%	NP	NP	33.6%	25.0%	9.6%	19.7%	32.4%	NP	32.3%	32.6%	13.7%
Idaho	38.9%	55.7%	17.7%	NP	NP	NP	17.4%	55.0%	15.4%	16.6%	48.0%	NP
Illinois	24.2%	62.4%	26.7%	29.6%	51.5%	13.4%	10.8%	63.5%	26.0%	10.9%	72.2%	36.8%

References posted on www.devmathrevival.net

More "Nowhere" Data

GRADUATION RATES OF FULL-TIME STUDENTS ENROLLING IN REMEDIAL EDUCATION

	Certificates			Associate degrees		
	Entry cohort, started fall 2005	In 2 years		Entry cohort, started fall 2004	In 3 years	
Arizona	NP	NP	NP	NP	NP	NP
Arkansas	389	78	20.1%	4,219	396	9.4%
California (CSU system only)	NP	NP	NP	NP	NP	NP
Colorado	DS	DS	DS	3,061	281	9.2%
Florida	442	40	9.0%	19,413	1,818	9.4%
Georgia	2,065	330	16.0%	5,701	410	7.2%
Hawaii	21	DS	DS	1,852	191	10.3%
Idaho	200	36	18.0%	635	81	12.8%
Illinois	1,080	98	9.1%	12,891	1,806	14.0%

Analysis of “Nowhere”

- Mis-use of statistics (used to support pre-determined positions)
- Some validity in basic message

- Good point: “College students come to campus for college, not more high school. Let’s honor their intentions — and refocus our own good intentions to build a new road to student success.”

Some Better Data

- Community College Research Center: "Regression Discontinuity" to estimate effect of developmental math for students close to the college cutoff
- Three large studies used. Results in Dev Math:
  2 null (no effect)
  1 negative (dev math led to lower college level pass rates for similar placement scores)
- Unstated: results are based on the impact of Intermediate Algebra (or lack thereof)

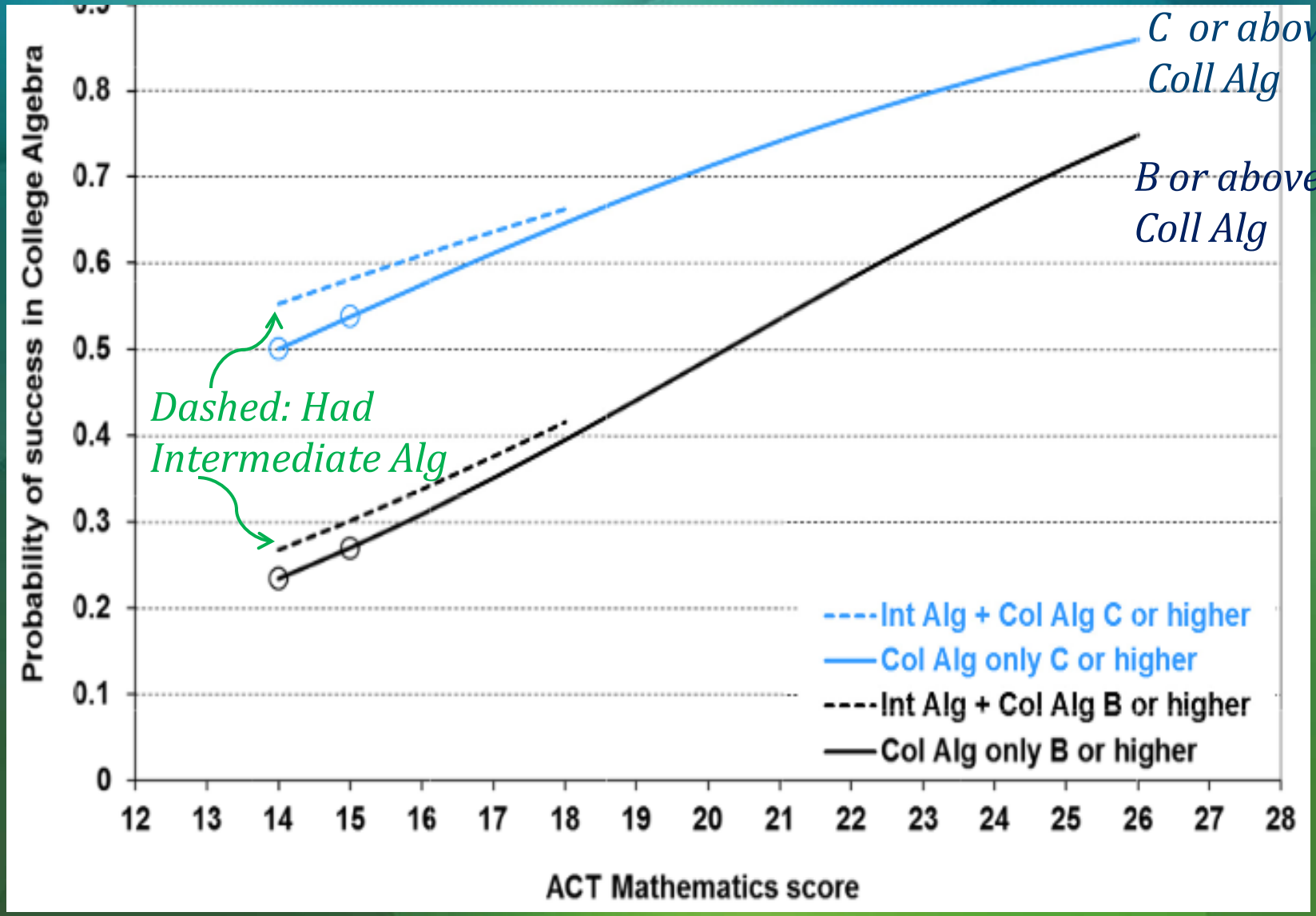
Regression Discontinuity

- Examine students with scores just below cutoff (dev math) and those just above cutoff
- **Developmental math should create a 'discontinuity' (improved outcome)**
- Sample data: Success rates in college math by score on placement test

Some Even Better Data: Intermediate Algebra → Coll Algebra

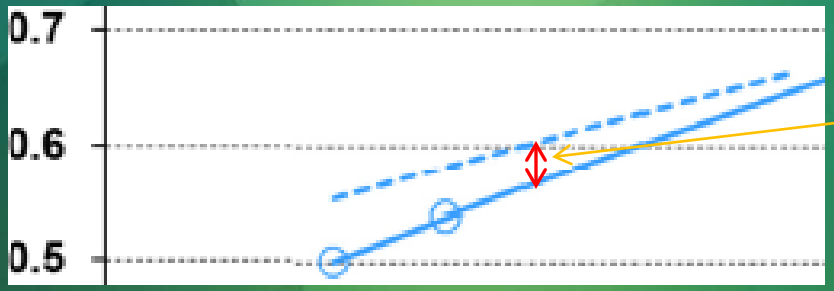
- ACT research study: 75 institutions, over 100,000 students
- Regression Discontinuity methodology
- Intermediate algebra increased probability of passing college algebra by 0.02 to 0.05
- Statistically significant … worth a semester?
- See graph on next page

ACT Report: Progression to Coll Alg



Summary of ACT Report

- Intermediate algebra effect: gain 2% to 5% pass rate in college algebra (CAlg)



Red line shows the 'effect size' of intermediate algebra

- By grade in intermediate algebra:
  Only A grades led to increased pass rate in CAlg
  B grades resulted in 'no improvement' (null)
  C grades associated with DECREASED passing

Explaining these results: Our Relics

- Intermediate Algebra as “Algebra II”
- Never designed to lead anywhere
- Intermediate Algebra is an accidental path to “College Algebra”
- Archeological value only **(see next ‘page’)**
- College Algebra never designed to lead anywhere (though that’s a separate talk ☺)

Intermediate Algebra as Algebra II

- Intermediate Algebra ⇐⇒ Algebra II from 1965-70
- School mathematics of that era: repetition over theory
- Designed so ‘anybody’ can teach it
- Very different from modern standards (NCTM, Common Core)

Engineering a Math Course

- Document mathematical needs
- Pre-calculus and calculus
- Science (college-level)
- Technical programs (mid- and high-skill)

Sources of Info: Math Needs

- "The Vision" project AMATYC with MAA: voices of partner disciplines
- MAA Curriculum Guide
- MAA CRAFTY College Algebra
- MAA CRAFTY Biology
- AMATYC Standards (Crossroads; Beyond Crossroads)
- AMATYC “Right Stuff” College Algebra

References posted on www.devmathrevival.net

The Pedigree of Algebraic Literacy

- The Algebraic Literacy course builds on decades of professional work
- Consistent with emerging research on learning mathematics at deeper levels
- A focus on good mathematics
- Content of which we can be proud

Algebraic Literacy Content Overview

See the green page of the handout.

- Numbers and Polynomials
- Functions
- Geometry and Trigonometry
- Modeling and Statistics

Core outcomes listed in each group

STEM-boosting outcomes listed (➔ pre-calc.)

Learning Outcomes posted on www.devmathrevival.net

Numbers and Polynomials

- Use of parameters and variables, including appropriate replacement sets
- Show procedural fluency with polynomial expressions, including basic factoring
- Use equations, inequalities, and systems of equations & inequalities
- Use exponential and power equations to represent situations
- Use symbolic procedures to manipulate formulas and literal equations
- STEM: radical expressions, rational expressions, additional factoring

Functions

- Understand basic algebraic functions – linear, exponential, and power
- Identify and write the appropriate function for a situation or set of ordered pairs
- Discrete or continuous models
- Understand properties of basic algebraic functions
- STEM: radical functions, rational functions, conic sections

Geometry and Trigonometry

- Use properties of basic geometric shapes
- Understand the patterns of measurement – perimeter, area and volume
- Understand the three basic trigonometric functions in the context of right triangles
- Use the three basic trigonometric functions

Statistics and Modeling

- Use basic concepts of measurement and data
- Understand theoretical and modeled relationships
- Use technology to generate models
- Understand how to judge which model is a better choice

Side-by-side

Algebraic Literacy	Intermediate Algebra
Designed to prepare students for key targets	Derivative of algebra II; not designed to prepare
Understanding mathematical systems; symbolism and application	Heavily symbolic and procedural; applications based on patterns and 'recipes'
Fewer procedural prerequisites (accessible)	More procedural prerequisites (barriers)
Content drawn from multiple mathematical domains	Content overwhelmingly algebraic

See "How to Recognize an Algebraic Literacy Course" on www.devmathrevival.net

Benefits of Algebraic Literacy

- Content based on student need
- Fits multiple math paths
- Shorter course sequences: shorter paths (acceleration)
- More context, increased focus on reasoning skills
- Adapts to local needs

Sample Lessons for Algebraic Lit

- Documents are “Creative Commons” license (by attribution)
- Trig Basics 2.x
- Rational Exponents 3.x
- Rate of Change (exponential) 4.x

Sample lessons available at www.devmathrevival.net

A sample lesson (2.x)

Lesson 2.x – Trigonometric Functions (basics)

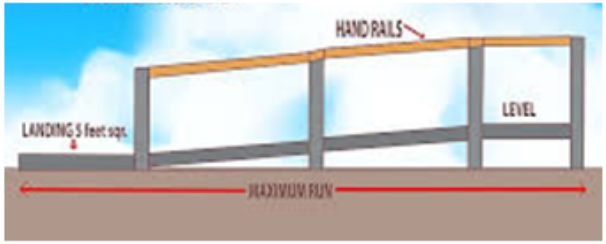
When we looked at geometric shapes, we measured the size of angles by degrees – 90 degrees is a right angle, for example. For comparison, a 5 degree angle is 'small'. Perhaps you have wondered how the lengths of the sides for a degree relate to the number of degrees. Trigonometry will help us find answers.

We will start by looking at two similar right triangles

Reminders: 360 degrees is a complete 'revolution'. Ramps need to have a small slope for safety.

PART A:

A ramp for wheel chair access needs to have an angle of 5 degrees.



Here is the sloped portion of the ramp

Here is the ramp to the first support:

Both of these portions of the ramp are right triangles. We might think we need to

A sample lesson (3.x)

Lesson 3.x – Understanding Rational Exponents

Earlier, we used integer exponents for a number or variable base, like these:

4^3 $3x^{-2}$ $25n^3$

Positive exponents indicate a repeated product $25n^3 = 25 \cdot n \cdot n \cdot n$

Negative exponents indicate a division by a repeated product

$$3x^{-2} = 3 \cdot \frac{1}{x^2}$$

Separate from those situations, we saw a constant base with a variable exponent, like $y = 2^x$

Perhaps you wondered about the exponent values (x) that are fractional or decimal numbers. What does $2^{\frac{4}{5}}$ mean? How about $2^{1.6}$?

Part A:

As we often do, we will start with something we know: $4^3 = 64$

We also know that a whole number is equivalent to a fraction: $3 = \frac{3}{1}$

This expression includes parentheses around the fractional exponent $\left(\frac{3}{\ }\right)$.

A sample lesson (4.x)

Lesson 4.x – Rates of Change and Health

We've seen situations where there is a constant 'adding' rate of change, and we know that this type of change is related to the general linear model $y = mx + b$. In our work with this model, the rate of change has been called the slope of the line. The rate of change is the same everyplace on the line. [If the rate of change varied, the graph would not be a straight line!]

Additive change means a linear equation $y = mx + b$

The slope is the rate of change

Reminder: Rate of change is "output changes this amount per 1 unit of input change".

We've also looked at situations where change is based on multiplying, and the exponential model. This lesson involves exponential patterns … and others.

Part A:

Infection diseases are tracked by specialists who are interested in predicting how many people will become ill. One recent infection had two characteristics: Each infected person passed along the infection to two other people, and the number infected grew by 5% each day.

Disease specialists use both kinds of information (per person, and per day). One tells us how contagious the disease is (number infected per person); the other estimates the number infected over time.

First, here is a chart of the number infected starting with one person. The 'step' listed is not a time value like a day; the step represents people passing along the infection …

Related Info

- The Algebraic Literacy course is from the AMATYC New Life Project (Dev Math Comm)
- The Dana Center New Mathways project has a path similar to AL ("Reasoning with Functions")
- New Mathways focuses on state or district implementation
- AMATYC New Life: focus on faculty; adapting to local conditions
- Carnegie Foundation Pathways project is piloting a 'bridge' course

References posted on www.devmathrevival.net

Summary: Algebraic Literacy

- Good mathematics … designed to work
- Effective preparation
- Connected … diverse … reasoning
- Not just a new book, not just ‘flipped’
- Replace ineffective intermediate algebra
- Based on professional work: sustainable and scalable

Ending stuff

- Other Questions?
- Contact information on the references
- “Math Literacy Outcomes” on www.devmathrevival.net
- Join the “New Life” work: See the references
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